DATA COMPRESSION

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THESIS

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ABSTRACT

A study of data compression techniques involving linear interpolation and linear prediction showed that redundancy is a problem that can be significantly reduced by various polynomial approximations. A more recent compressor, the continuous secant compressor which determines the optimum sampling interval prior to sampling, was found to be the most efficient compressor examined. The continuous secant compressor bases its reduction technique on a straight-line approximation. Data compression results when the system in question does not occupy its entire bandwidth. The addition of white noise over the entire bandwidth was found to reduce the efficiency of the continuous secant compressor by only a small amount. The probability distribution of the straight-line approximation in the presence of noise had a gaussian distribution and a relatively small standard deviation.

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I. INTRODUCTION

A. DATA COMPRESSION

Millions of dollars are being spent every year storing, processing and transmitting repetitious video and telemetry data. [Ref. 1] As a result of research in such fields as space exploration and television, there is an increasing demand for a more efficient means for handling this redundant data. The growth of data from telemetry systems has increased exponentionally in the past few years and is expected to increase even more in the future. Much concentrated effort has been done in designing a fairly simple and reliable means of transmitting only the significant changes in data instead of processing all the generated data. This research has produced a number of techniques which make up the field of data compression. For many years this extraction of significant data had been accomplished by human analysts. Today data compression is being simulated by electronic devices but the implementation of these techniques has not been thoroughly established, even though it is gaining significant recognition as the technology which can: (1) reduce physical and power requirements for a spacecraft system, (2) increase ground communication data flow and (3) reduce computer costs and data storage.

A good application of a data compressor and its possible attributes would be to a pulse-code-modulated(PCM) telemetry system, from a spaceborne sensor to the user on the ground [Ref. 2]. The data compressor could be incorporated into either the spaceborne transmitting system or the ground receiving system. Diagrams of both possibilities appear in Fig. 1.1. The advantages of a data compressor in a space vehicle are as follows: (1) By reducing the number of samples from all the generated samples the transmitter power or transmitter bandwidth may be reduced; (2) By maintaining the same transmitter power more samples can be transmitted, thus increasing the information rate; and (3) The operating range can be increased due to the reduced system bandwidth. By placing the data compressor in the ground system the following benefits could be achieved (These also apply to the spaceborne system.): (1) Data compression could allow for a reduction in electronic processing equipment; and (2) Data compression could allow a given amount of information to be represented by fewer symbols which in turn reduces the time required for processing the data.

B. CLASSIFICATIONS OF DATA COMPRESSORS

Data compressors may be classified into four basic categories. The complete classification is shown in Fig. 1.2 [Ref. 3]. Parameter extraction reduces the bandwidth required to transmit a given sample by means of an information-describing irreversible transformation [Ref. 4]. Examples of parameter extraction techniques are phase comparitors, spectrum analyzers and peak detectors. A second method is adaptive sampling which is the technique that synchronizes the sampling rate with the rate of data activity. The third method, encoding, transforms a message into coded words. Examples of encoding techniques include delta modulation and probability descriptions. The last category is the primary subject of this thesis, <u>redundancy reduction</u>. Redundancy

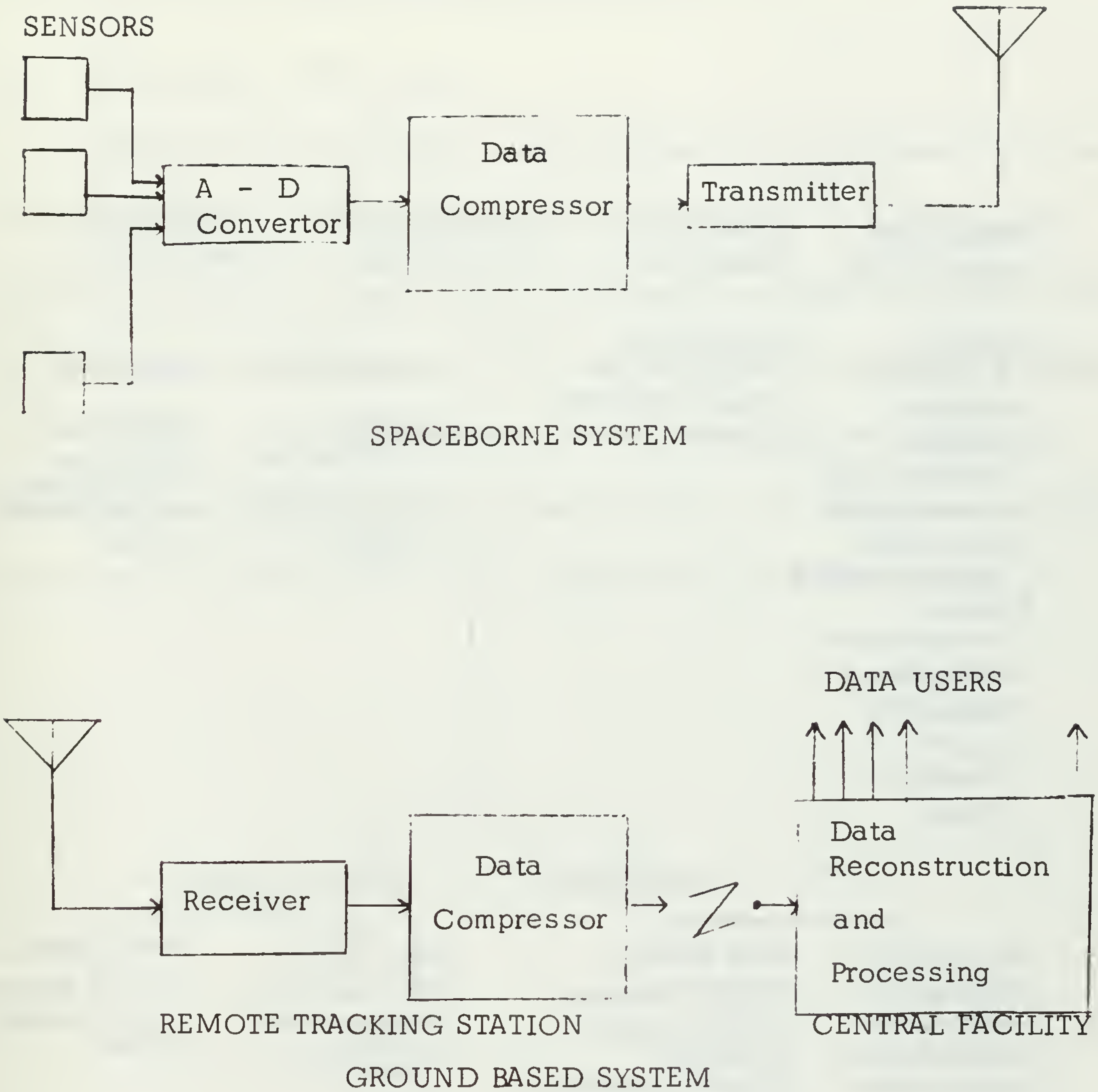

FIGURE 1.1 AN EXAMPLE OF HOW A DATA COMPRESSOR IS USED IN A TELEMETRY SYSTEM

- PARAMETER EXTRACTION
  - Spectrum Analysis
  - Filtering
  - Fourier coefficients
  - Thresholding
  - Frequency discrimination
  - Phase comparison
  - Histograms
  - Decoding
  - Pulse Shaping
  - Other
- ADAPTIVE SAMPLING
  - Command Controlled
  - Self-adaptive
  - Other
- REDUNDANCY REDUCTION
  - PREDICTORS
    - PEAK ERROR
      - Polynomial
      - Sine Wave
      - Exponential
      - Non-linear
      - Other
    - RMS ERROR
      - Least Squares polynomial
      - Wiener
      - Eigen Functions
      - Cyclic patterns
      - Other
  - INTERPOLATORS
    - PEAK ERROR
      - Polynomial
      - Exponential
      - Continuous Secant
      - Other
    - RMS ERROR
      - Least Squares polynomial
      - Other
- ENCODING (STATISTICAL)
  - NONADAPTIVE
    - Delta Modulation
    - Incremental
    - Probabilistic
    - Other
  - ADAPTIVE
    - Bit Plane
    - Adaptive coefficient
    - Probabilistic
    - Other

FIGURE 1.2 CLASSIFICATIONS OF DATA COMPRESSION reduction eliminates redundant samples by comparing previous or succeeding samples with arbitrary reference patterns.

C. REDUNDANCY REDUCTION

Shannon defines redundancy as "that fraction of a message or datum which is unnecessary and hence repetitive in the sense that if it were missing the message would still be essentially complete or at least could be completed."[1] There are two basic methods employed under the heading of redundancy reduction: prediction, which determines redundancy by comparing predicted samples to actual samples; and interpolation, which determines the redundancy of past samples prior to transmission.

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[1]Kortman, C. M., "Data Compression by Redundancy Reduction," IEEE Spectrum, v. 4, p.134, March 1967.

II. SYNCHRONOUS SAMPLING

A. INTRODUCTION

Data compression techniques can be divided into two classes; those which destroy the time reference and transmit the significant samples at a constant rate, and those which transmit only significant samples as they occur in time. The first of these methods is termed synchronous sampling. Due to the fact that the interval between samples is not constant, additional information is needed to identify the time base. This additional information must be applied to digital words and therefore cannot be implemented on analog input signals.

Synchronous compressors have been simulated basically for television bandwidth compression studies. Many of the techniques used in prediction and interpolation have been simulated for television bandwidth compression [Refs. 5, 6, 7, 8]. Interpolation and prediction techniques have been applied to video information with the first-order interpolator giving the best results, yielding compression ratios in the order of two to four [Ref. 7]. The compression ratio is defined as:

$$CR = \frac{\text{Total number of samples}}{\text{Number of significant samples}} \qquad (1\text{-}1)$$

B. CHANGE TRANSMISSION

If the transmitter and the receiver use identical prediction techniques, only changes from the predicted values need be transmitted in order to reconstruct the data set, rather than transmitting the values themselves. A high sampling rate would give small changes in the signal thus reducing the amount of information to be transmitted; but for an input signal with significant changes between samples, the compressor would have little or no effect on the amount of data to be transmitted.

C. VARIABLE SAMPLING RATE

A significant improvement in system efficiency can be obtained by sampling at higher rates during high data activity and at lower rates during relatively inactive periods. In order to employ this type of a compressor in a system, the periods of high and low data activity must be known a priori, or a timing mechanism must be incorporated into the compressor which would act as a switch for varying the sampling rate.

A sporadic input signal could put serious restrictions on the system. The efficiency of such a system would largely depend upon the hardware employed in the switching circuit. A constantly changing signal would require a constant high sampling rate in order to reproduce the signal with any degree of accuracy. Data compression results from not transmitting during relatively inactive periods of signal activity; therefore, if the signal contains no inactive periods every sample will be transmitted and data expansion could result.

D. DELTA MODULATION

A continuous signal can be reconstructed by transmitting a series of pulses of equal width and magnitude that differ only in sign [Ref. 9]. At each succeeding sample point the change in the input signal is approximated by adding or subtracting these pulses to the previous data point. Data compression results from transmitting only the series of pulses or changes in the signal. The basic restriction in delta modulation is similar to the problem that occurs in change transmission; a high sampling rate is needed to reconstruct high-frequency signals which may result in data expansion.

III. PREDICTORS

A. INTRODUCTION

The prediction technique requires that one have knowledge of past samples. This a priori knowledge can be obtained from either previous experiments or from recent samples generated by the experiment in question. The history of past samples used to predict a time-varying function is limited only by the hardware employed in the predictor and the predictor efficiency. Evidence has shown that a complex system need not be the most efficient. Studies have shown that polynomial methods are extremely efficient and fairly easy to implement. The block diagrams of a simple predictor used as a data compressor are shown in Fig. 3.1 and Fig. 3.2 [Ref. 2, 10].

The predictor estimates the value of the next sample by comparing previous data samples, using a predefined method. If the difference between the predicted sample and the actual sample is within certain error limitations placed on the system, then the sample is not transmitted because it is redundant. Prediction techniques can be linear or non-linear.

Linear prediction techniques assert that the predicted sample will lie on an nth-order polynomial. This concept can be described by the following equation [Ref. 2];

$$X_t = X_{t-1} + \Delta X_{t-1} + \Delta^2 X_{t-1} + \ldots\ldots \Delta^n X_{t-1} \qquad (3\text{-}1)$$

where

X_t = predicted sample value at time t

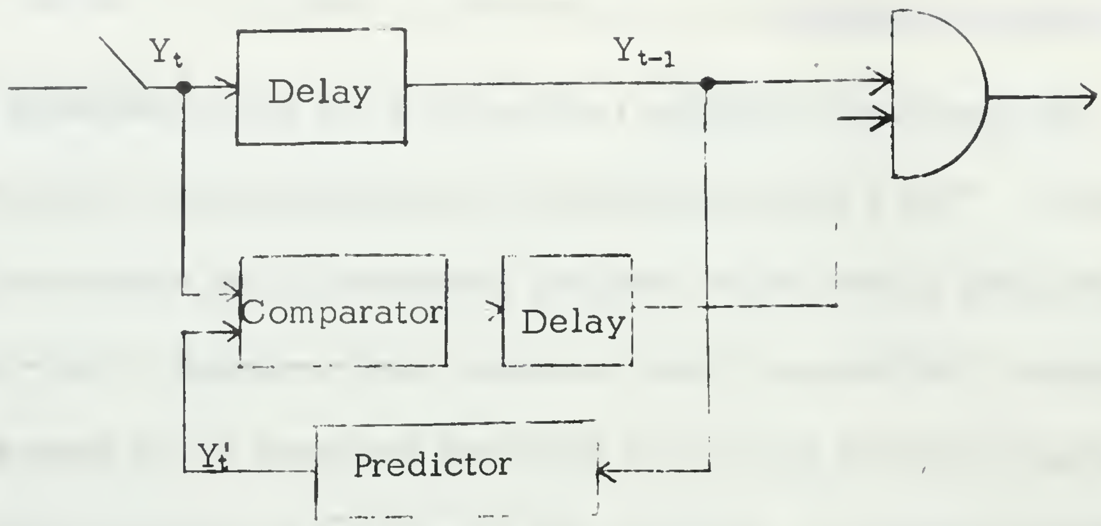


FIGURE 3.1 A BASIC ASYNCHRONOUS PREDICTOR

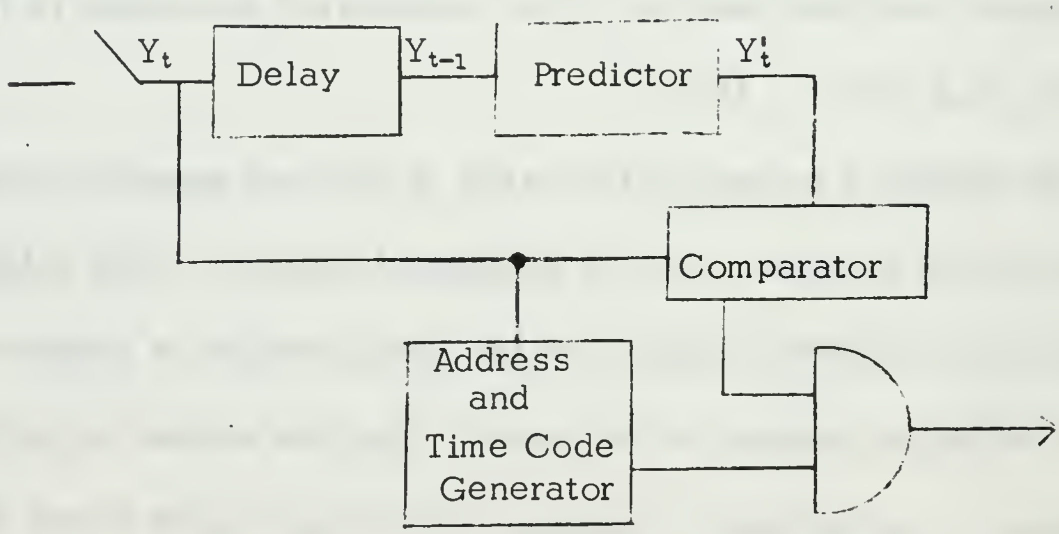


FIGURE 3.2 A BASIC SYNCHRONOUS PREDICTOR

X_{t-1} = previous data sample at time t-1

$$\Delta X_{t-1} = X_{t-1} - X_{t-2}$$

$$\begin{aligned} \Delta^2 X_{t-1} &= \Delta X_{t-1} - \Delta X_{t-2} \\ &= (X_{t-1} - X_{t-2}) - (X_{t-2} - X_{t-3}) \\ &= X_{t-1} - 2X_{t-2} + X_{t-3} \\ &\vdots \\ \Delta^n X_{t-1} &= \Delta^{n-1} X_{t-1} + \Delta^{n-1} X_{t-2} \end{aligned}$$

B. ZERO-ORDER PREDICTORS

The simplest predictor is one in which a constant value is to be monitored. A good example of a constant value which could be incorporated into a data compression system is the temperature of a space capsule, which need not be constantly monitored if an efficient data compressor is used. The temperature would be transmitted only if the change was significant. For the zero-order predictor n=0 and equation (3-1) reduces to

$$X_t = X_{t-1} \tag{3-2}$$

which means that the predicted sample should be identical to the previous sample. To implement a constant value predictor, all that is needed is a delay element and a comparator. If for the constant-value predictor, the constant value were to increase or decrease slowly, then the errors would go undetected, and the system could go unstable. To avoid this continuous error from the redundant samples X_{t-1} refers only to the last transmitted sample.

The predictor which predicts that each sample will be the same as the previous transmitted sample is called a zero-order predictor

[Ref. 2, 3, 4, 9, 10, 11]. The fixed-aperture zero-order predictor compares each sample to the limits of the predefined error. If the sample lies outside these limits then it is transmitted and the next sample is compared to a new set of limits. The new error limits are the original limits shifted so that the new upper limit is the original lower limit or the new lower limit is the original upper limit. The direction of the shift depends on whether the sample exceeded the original lower bound or the original upper bound. The use of the new sample as the basis for succeeding samples is termed floating-aperture prediction. It is similiar to the fixed-aperture zero-order predictor except that the tolerance limits are placed about each significant sample as it occurs. Both zero-order predictors predict that each sample will be the same as the preceeding sample, or in other words, they try to fit the data with a horizontal straight line. The floating-aperture and the fixed-aperture predictors are shown in Fig. 3.3 and 3.4 respectively.

There is one other zero-order predictor which is a version of the floating-aperture predictor where a priori knowledge of an established trend is used to make the data compressor more efficient. The difference between this zero-order predictor and the previous floating-aperture zero-order predictor is that each sample is offset by a predetermined amount, the sign of the offset being determined by the sign of the most recent deviation of the predicted sample from the actual sample. This zero-order offset predictor is shown in Fig. 3.5 [Ref. 2].

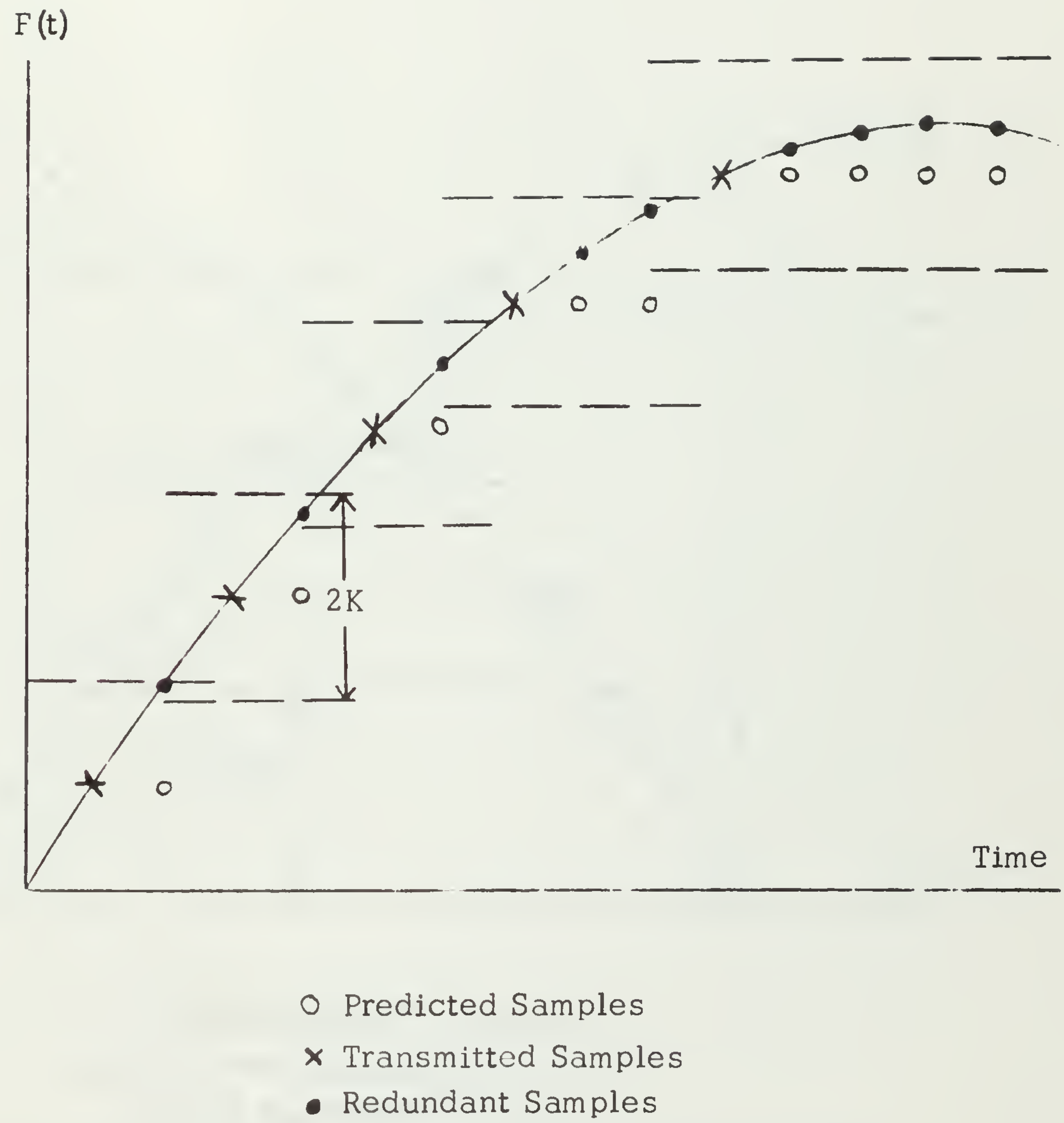


FIGURE 3.3 ZERO-ORDER FLOATING-APERTURE PREDICTOR

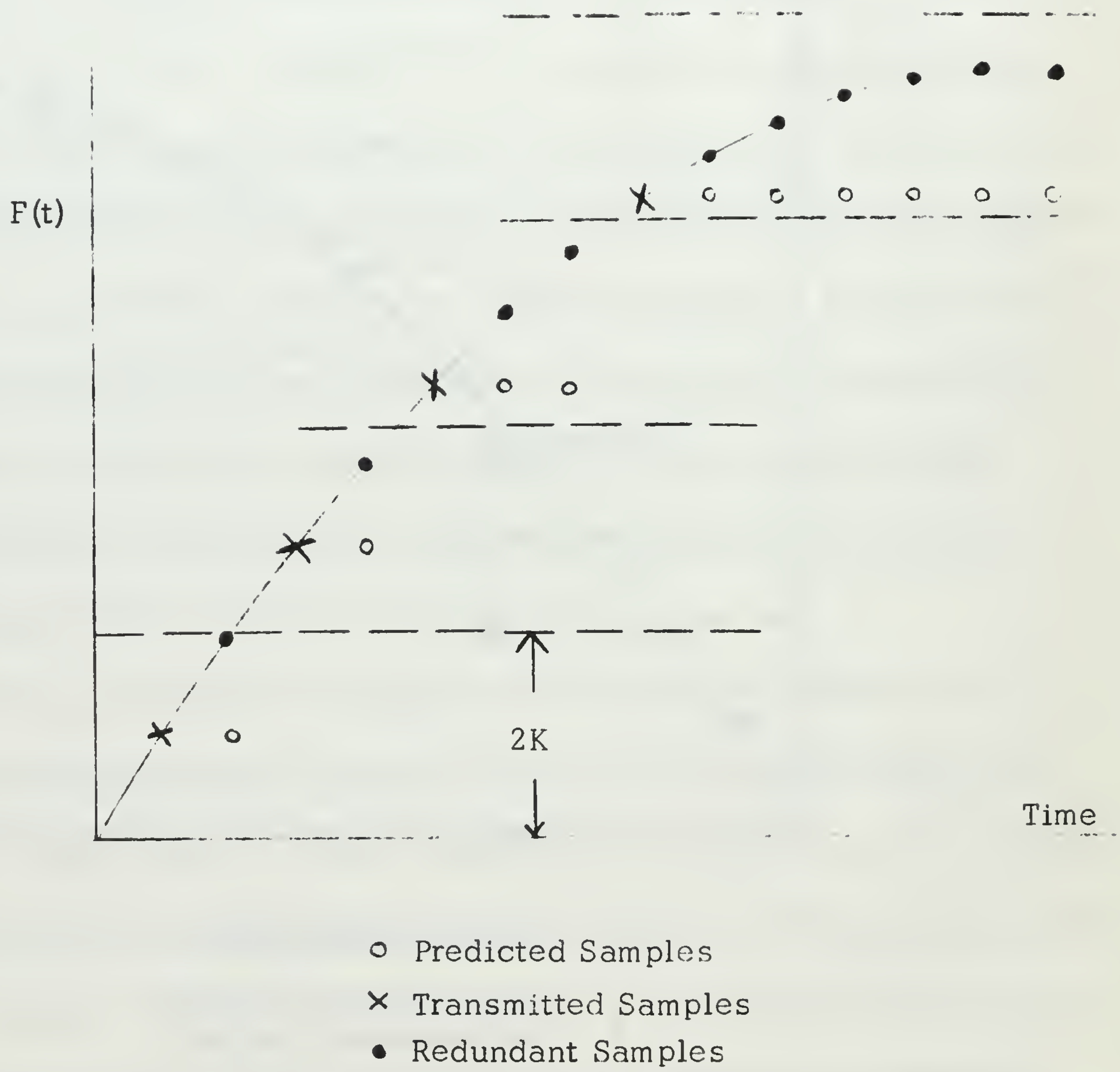


FIGURE 3.4 ZERO-ORDER FIXED-APERTURE PREDICTOR

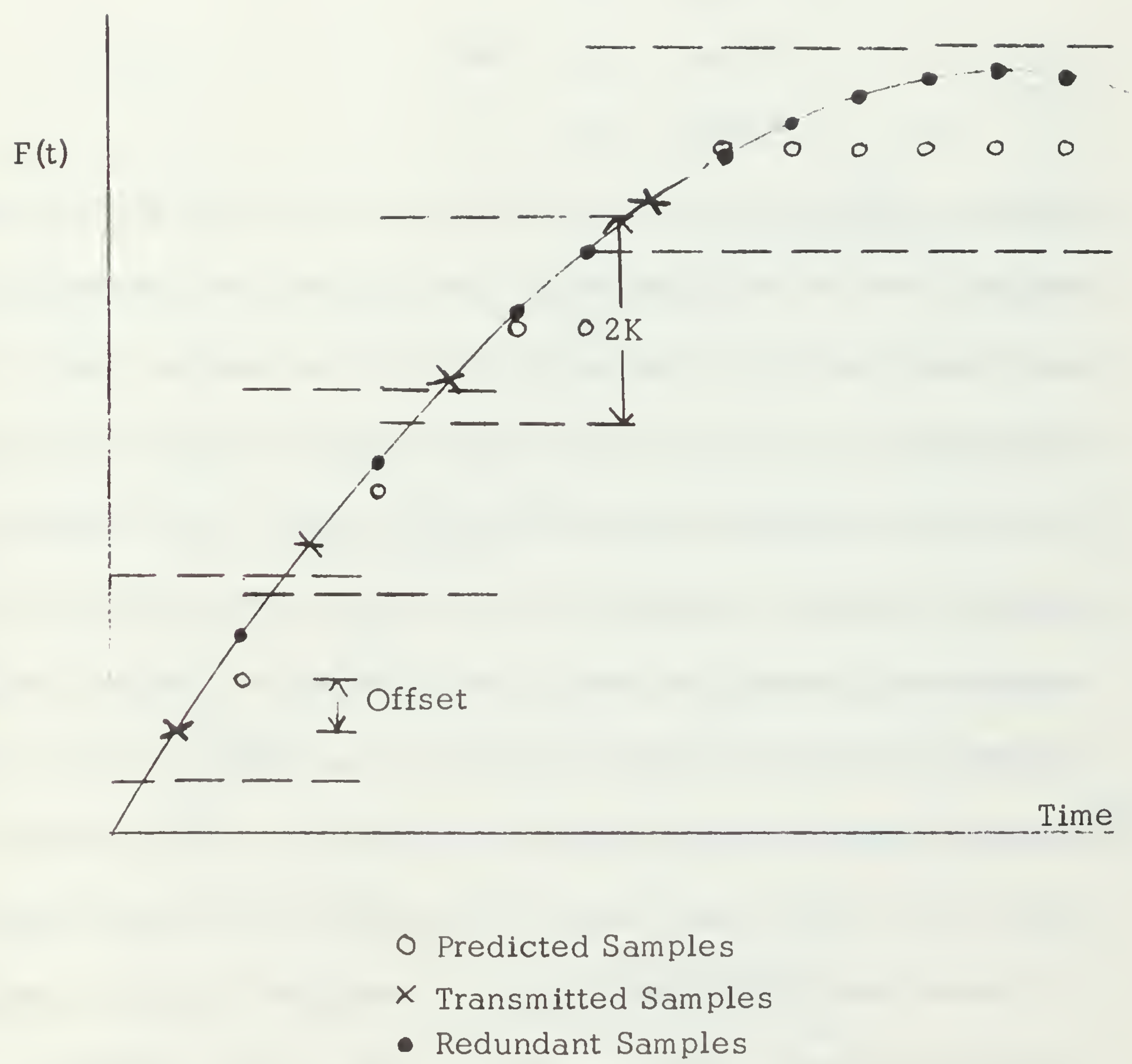


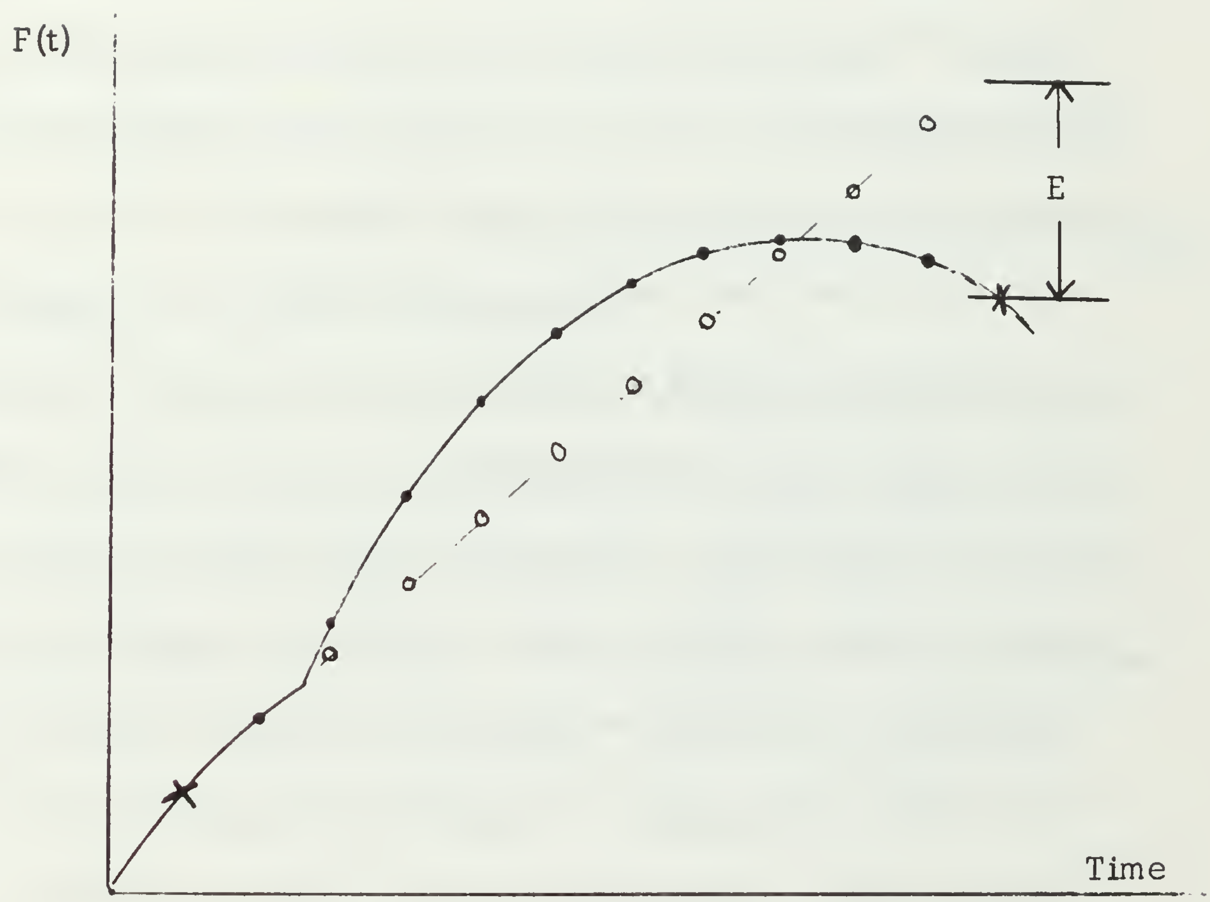
FIGURE 3.5 ZERO-ORDER OFFSET PREDICTOR

C. FIRST-ORDER PREDICTOR

The last predictor to be examined is the first-order predictor [Ref. 2, 9, 13]. As the name implies, the first-order predictor predicts that each succeeding sample will be the same as the preceeding sample plus the same change that occurred between the two preceeding samples. For the first-order predictor equation (3-1) becomes

$$\begin{aligned} X_t &= X_{t-1} + \Delta X_{t-1} \\ &= X_{t-1} + (X_{t-1} - X_{t-2}) \\ &= 2X_{t-1} - X_{t-2} \end{aligned}$$

Since X_{t-1} represents the change that occured between the two previous samples, then the predicted sample becomes the previous sample plus the same change that occured between the previous samples. As in the zero-order predictor, in order to avoid errors from the redundant samples, X_{t-1} or X_{t-2} or both must be the transmitted samples rather than the redundant samples. In graphic terms the first-order predictor would be implemented by drawing a straight line between two samples and then predicting that the next sample will fall on an extension of this straight line within specified error limitations. The first-order predictor is shown in Fig. 3.6. If the actual sample falls outside the tolerance limits then it is transmitted and the next sample is predicted to lie on a straight line connecting the previous two samples. As long as the samples remain redundant, or within the specified tolerance limits, then the straight line is extended indefinitely until a sample that falls outside the tolerance limits occurs.



○ Predicted Samples

× Transmitted Samples

● Redundant Samples

FIGURE 3.6 FIRST-ORDER PREDICTOR

D. CONCLUSIONS

It follows that the second-order predictor would go back one more sample and make allowance for the change in the change from the previous data samples. The system becomes very complex as n increases and for simplicity purposes only the zero-order and the first-order predictors were examined.

Although the zero-order predictor is the only data compressor in actual operation there are many shortcomings of predictors in general. The basic problem is that for high data activity almost every sample is transmitted which could result in data expansion, and for low data activity relatively few samples are transmitted. This means that the transmitter must be able to accept a very fast sample rate at a time t, and at a time $t+\Delta t$, the transmitter must be able to work efficiently at a very slow sample rate. Obviously this will require a fairly complex system. Another problem is that in order for a synchronous compressor to be efficient it must be sampled at a very high rate (i.e. For an error of one-tenth of one percent, the function must be sampled over 6,000 times the highest-frequency component in the signal [Ref. 9].) Prediction also tends to amplify noise so that during high or low data activity the prescribed error tolerence will not be constant. Although many authors give methods for obtaining the optimum linear predictor, they all agree that in general the power spectrum of the compressed signal is indeterminant before compression can be applied, and, therefore, by assuming a power spectrum a suboptimum predictor will result.

IV. INTERPOLATORS

A. INTRODUCTION

Prediction techniques are educated guesses based only on the assumption that the data will remain relatively constant from one time interval to the next. If the sampled data is randomly distributed or corrupted with noise, then the redundancy reduction efficiency of the predictor will be relatively low for acceptable accuracies. A higher compression ratio can be obtained by relying on future samples as well as past samples. This process of determining redundancy after the sample has been examined is termed interpolation.

Interpolation requires that a time-delay element be present in the system which might be difficult to implement in certain compressor systems. Interpolation uses present samples to determine where past samples should have been and compares this prediction to the actual position of the past sample. In some interpolators, the length of time delay needed is directly related to the number of redundant samples.

One advantage of interpolation over prediction is the increased signal-to-noise ratio which may be tolerated by interpolators. Most predictors require that the transmitted sample be used as the basis for the next prediction. If this transmitted sample is contaminated with noise then the predicted value is not accurate. Interpolators use the succeeding samples to determine redundancy; thus, the transmitted sample that was contaminated with noise has little effect on the approximation. Predictors amplify noise by predicting that the next sample will include added noise. As a result interpolation techniques are not as susceptible to system noise as prediction techniques.

The compression ratio is higher for interpolators than for predictors. Once again this is due to the fact that the sample is not transmitted until at least one succeeding sample is examined. This insures greater accuracy which results in more redundant samples, thus increasing the compression ratio. These advantages make interpolation the most desirable form of data compression, but the delay problem has impeded its progress.

B. ZERO-ORDER INTERPOLATOR

The first interpolator to be examined is the zero-order interpolator which is very similar to the zero order predictor [Ref. 10]. As in the zero-order predictor the redundant set of samples is represented by a horizontal straight line. The transmitted sample is not the actual sample but is the average of the most positive sample and the most negative sample in the redundant set. The transmitted sample can be expressed by the following;

$$Y_t = \frac{Y_1 + Y_u}{2}$$

where

Y_t = transmitted sample

Y_u = largest sample value in the redundant set

Y_1 = smallest sample value in the redundant set

The zero-order interpolator compares each sample to the preceeding samples in the redundant set, determining the largest and smallest samples in the set. When the point is reached where $Y_u - Y_l \geq E$, the predefined error, the sample Y_t is transmitted. The spread that can be tolerated in the zero-order interpolator is strictly dependent upon the predefined error. The zero-order interpolator is shown in Fig. 4.1.

C. FIRST-ORDER INTERPOLATOR

The other linear interpolator is the first-order interpolator [Ref. 9, 10]. Higher-order interpolators are very complex with a relatively small increase in the compression ratio [Ref. 9]. The first-order interpolator represents the redundant sample set by a straight line. There are two possibilities of representing a data set by a first-order interpolator. The first of these is the four-degree-of-freedom interpolator in which there is freedom of both the starting and ending points. The starting and ending points are computed so that the straight line will approximate as many samples as possible within the error tolerence limit. This process is also termed the Chebychev approximation. The four-degree-of-freedom first-order interpolator is shown in Fig. 4.2a. The computed straight lines can be connected by extending either the starting or ending points of successive straight lines. The two-degree-of-freedom first-order interpolator, shown in Fig. 4.2b, is the same as the four-degree-of-freedom first-order interpolator except that one end of the straight line is anchored whereas the other end is free to move, so that the line can represent the maximum number of samples. The predefined error is the limiting factor in both first-order interpolators. As shown in the figures, the four-

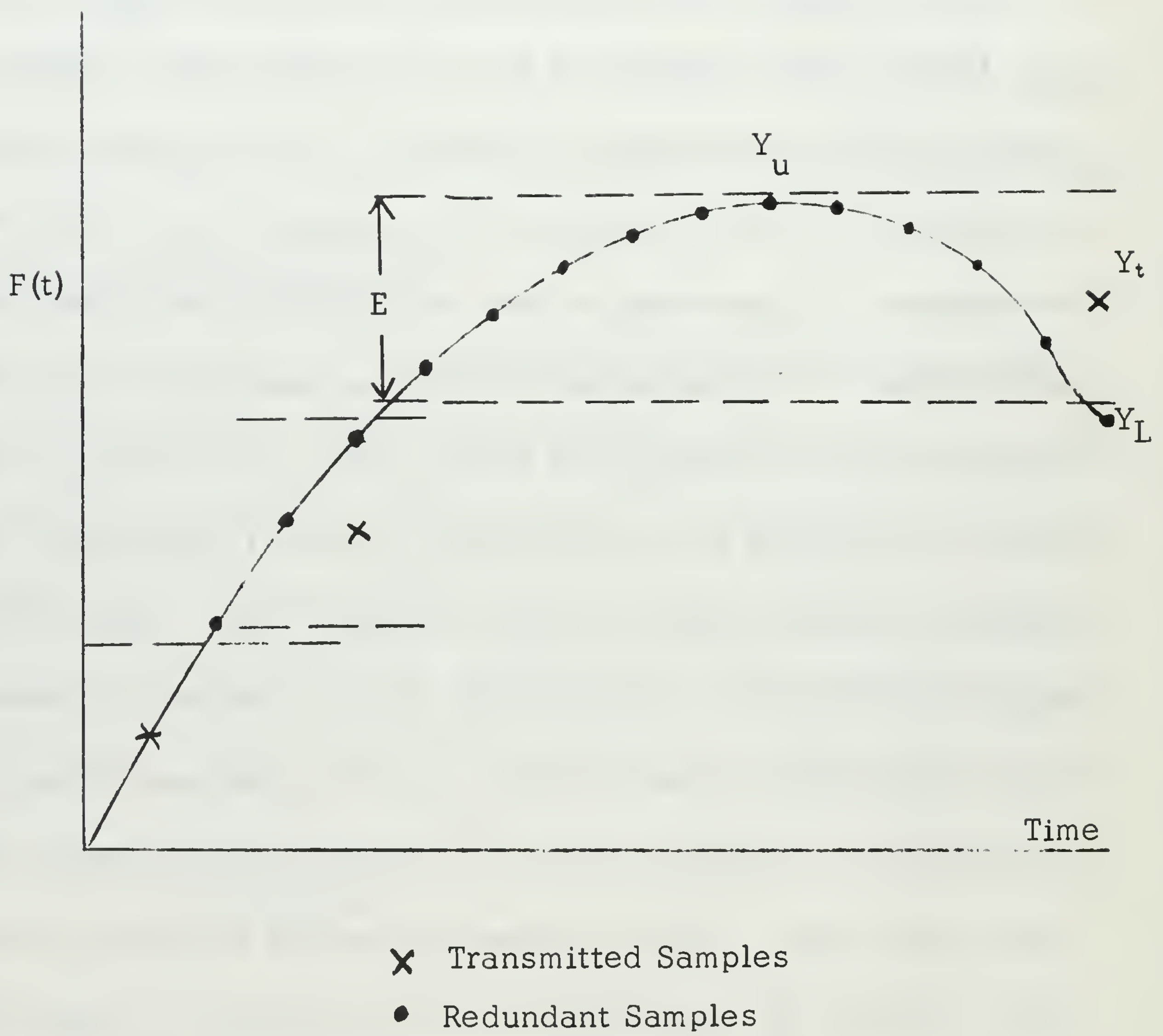


FIGURE 4.1 ZERO-ORDER INTERPOLATOR

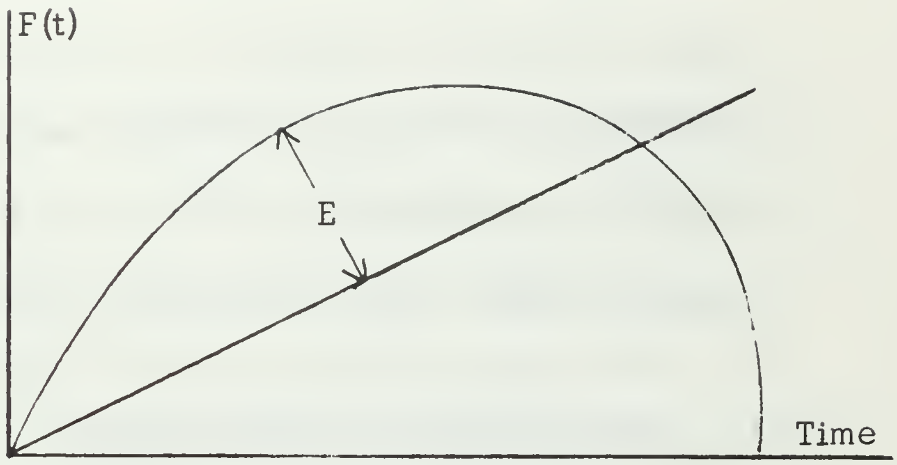


FIGURE 4.2b TWO-DEGREE-OF-FREEDOM INTERPOLATOR

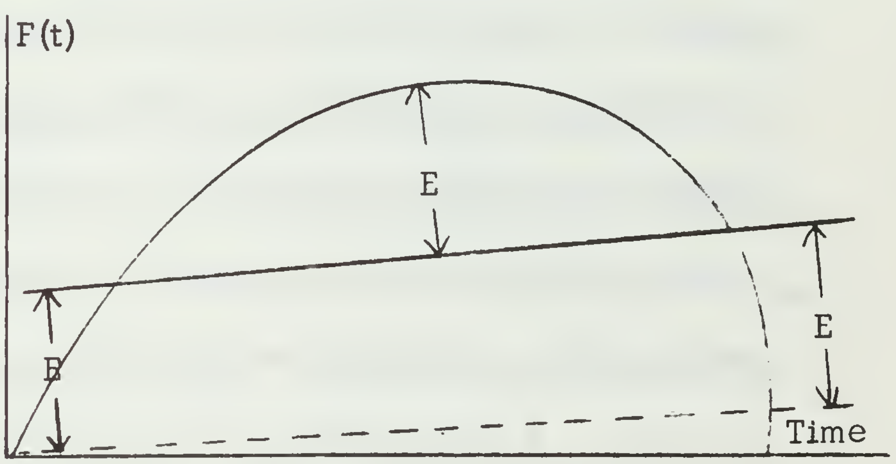


FIGURE 4.2a FOUR-DEGREE-OF-FREEDOM INTERPOLATOR

degree-of-freedom first-order interpolator can cover a wider range of signal that the two-degree-of-freedom first-order interpolator.

The two-degree-of-freedom first-order interpolator is less complex than the four-degree-of-freedom interpolator but since there is no justification in weighting some samples more heavily than others, the four-degree-of-freedom interpolator would give better redundancy reduction. One problem with these interpolators is that they require an enormous amount of computation time which could put serious restrictions on the computers which might be employed in the system.

D. FAN METHOD OF INTERPOLATION

Gardenhire proposed a method of interpolation which reduces the number of computations required for the above interpolators, but in essence produces the same results [Ref. 9]. This technique, termed the fan method of interpolation, operates similarly to the two-degree-of-freedom first-order interpolator. The calculations involve computing two slopes, both originating from the last transmitted sample, to the nth sample plus a specified tolerance and to the nth sample minus a specified tolerance. A line is then drawn between the last transmitted sample and sample n+1. If this slope lies between the first two slopes then sample n is redundant. If it is redundant then two new slopes are calculated from the last transmitted sample to sample n+2 plus and minus the tolerance limits. Then the slope to sample n+3 is compared to the new slopes and the old slopes. If this slope does not lie in the fan which is defined as the outer extremities of all the computed slopes, then the sample n+2 is transmitted and becomes the new starting point for succeeding samples. If this slope lies in the fan then the sample is redundant and the fan is extended by computing two new slopes. This process is repeated until a significant sample occurs. This method is shown in Fig. 4.3.

E. CONCLUSIONS

Since interpolators use the knowledge gained from past samples to determine redundancy, they have a distinct advantage over predictors which base their calculations on future predictions. Interpolation requires that the data compressor system incorporate a time delay to insure that samples are not transmitted immediately but delayed until succeeding samples can be examined. Predictors tend to amplify noise as each prediction is made, whereas interpolators would reduce this effect and would require a much lower signal-to-noise ratio. Predictors use only past transmitted samples as a basis for future predictions. If this sample contains noise, then this noise will be predicted to occur in the next sample. This pattern could continue at each succeeding sample, giving a net effect of noise amplification. This means that interpolators would be able to operate efficiently in a much noisier environment than predictors. The problems of increased complexity and time delay have impeded the progress of implementing an interpolator.

The data compressors were simulated on the SDS 9300 digital computer. The four-degree-of-freedom interpolator was not examined because of its complexity, but the two-degree-of-freedom interpolator was simulated. (Complete digital flow diagrams of all the data compressors

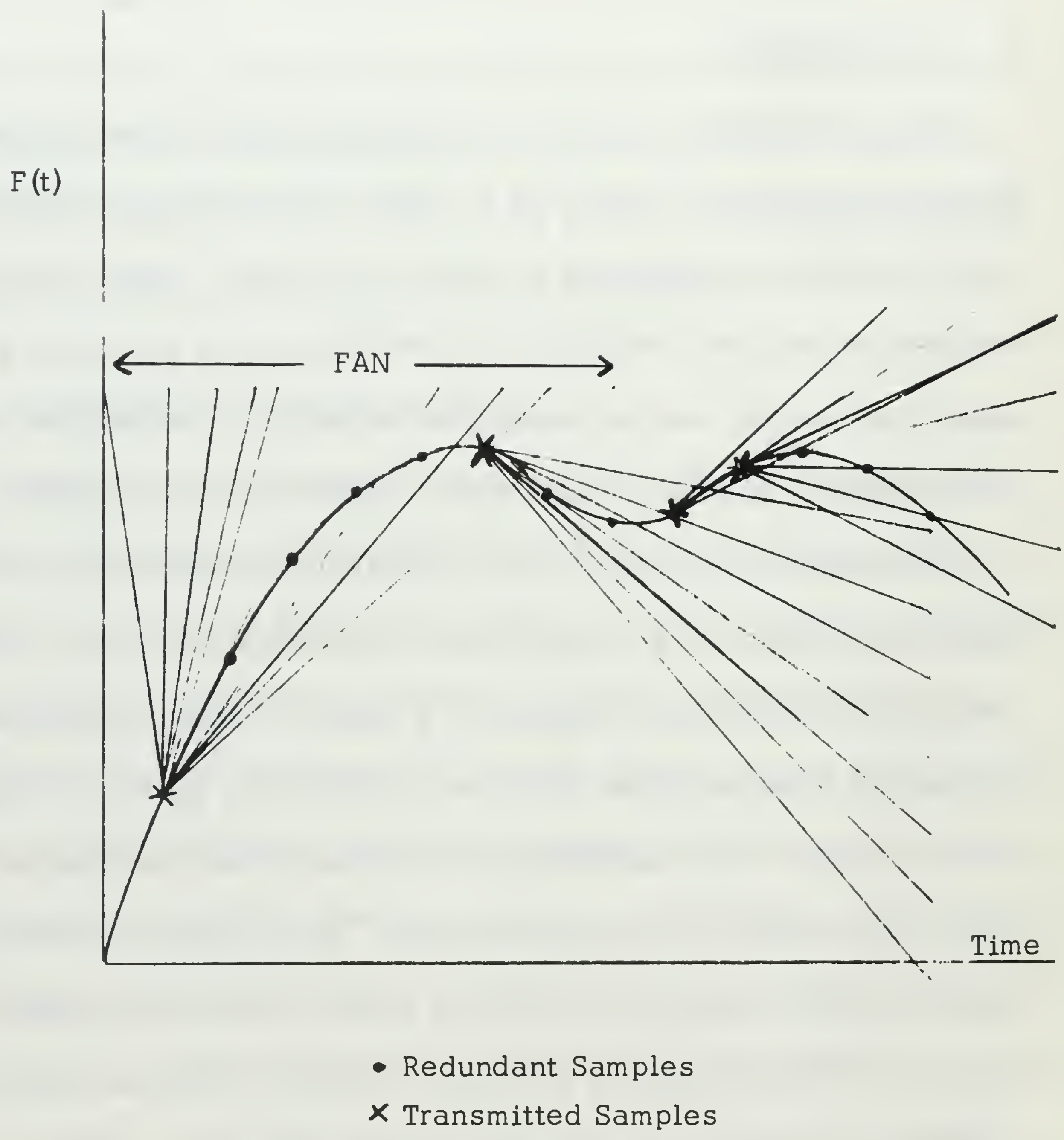


FIGURE 4.3 FAN METHOD OF INTERPOLATION

examined are contained in Appendix A.) Various input signals were simulated including a sine wave, an exponential and a random test input signal which is described in Chapter VI. The results of the test were based on the compression ratio versus the predefined error.

The results for the sine-wave simulation are shown in Fig. 4.4. The figure shows that for the predictors examined, the zero-order floating-aperture predictor has a lower compression ratio than the first-order predictor. This is not always the case. A simulation by Medlin on telemetry data showed that the zero-order floating-aperture predictor has a higher compression ratio than did the first-order predictor [Ref. 2]. This is one of the main reasons why the zero-order floating-aperture predictor has been the only implementation to date. The other predictors perform as expected with the zero-order fixed-aperture predictor having the lowest compression ratio. The same figure shows that all three interpolation techniques had a higher compression ratio than did any predictor. The basic reasons for this were explained earlier in this chapter. The performance of the fan method of interpolation was almost identical to that of the two-degree-of-freedom first-order interpolator.

The other two input signals gave similar results. The main difference was a shift in the curves. The curves shifted upward for the exponential input and shifted downward for the random input. The differences in information rate caused the observed shifts. Compression results when the input signal does not occupy the entire prescribed bandwidth. The random input signal does not allow for as much compression as does the

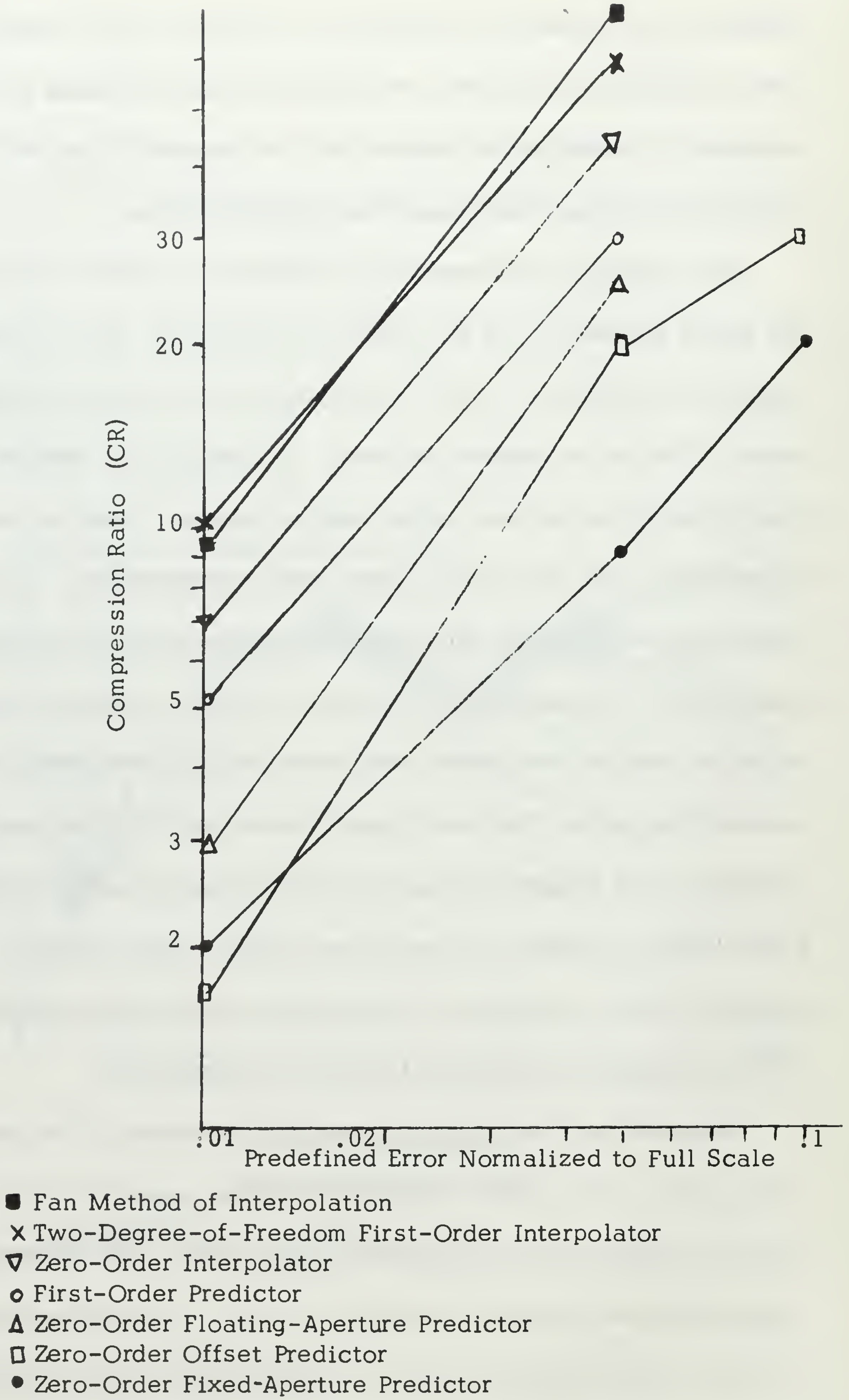


FIGURE 4.4 A COMPARISON OF DATA COMPRESSORS

exponential input because it occupies a greater percentage of its prescribed bandwidth.

The simulation showed that various types of data could represent a redundancy problem. Data compression seems to be a very effective means of reducing this redundancy problem.

V. CONTINUOUS SECANT METHOD

A. INTRODUCTION

Another technique for reducing redundant samples is to perform compression on the analog signal prior to sampling. This would ease the load of high data rates on digital computer computation and the amount of digital memory. This technique has been shown to be more efficient and it gives higher compression ratios than other methods previously described. Another advantage of reducing redundancy prior to sampling is that the sampling rate will be reduced thus easing the load on the sampling mechanism.

B. FORMULATION OF THE CONTINUOUS SECANT METHOD

The last data compressor to be examined is the continuous secant method of compression which will be presented much more detail than the previous data compressors [Ref. 9, 12]. Most signals of interest can be broken up into a series of concave or convex signals. The only assumption to be made in order to employ the continuous secant method is that the signal be continuous and have a fixed direction of concavity which changes a finite number of times in the continuous interval. This compressor, unlike the previous compressors that were simulated, acts on the continuous signal prior to sampling by determining the maximum interval between samples in which the function does not exceed a predefined tolerance level. This concave or convex interval is approximated by a secant such that the secant does not deviate from the continuous signal by more than the predefined maximum absolute error. Three slopes are needed in order to determine the optimum sampling interval. For the strictly concave case, $M_t(t)$ is the slope of the line connecting the continuous signal F(t) and the point A where A represents the beginning of the interval. The continuous secant method of interpolation is shown in Fig. 5.1. In equation form,

$$M_1(t) = \frac{F(t) - (F(t_0)+E)}{t-t_0} \tag{5-1}$$

where

t_0 = beginning of the interval

t = maximum sampling interval

E = predefined error

The second slope needed for the concave case is the line connecting the signal with point B as seen in Fig. 5.1; or in equation form,

$$M_2(t) = \frac{F(t) - F(t_0)}{t-t_0} \tag{5-2}$$

From Fig. 5.1 it is obvious that $M_1(t)$ is the maximum value of the slope from point B to the signal, F(t). This slope is equal to the slope of the tangent at the point T_1 [Ref. 9]. Equating slopes, $M_2(T)$ = Max $M_1(t)$, for $t>t_0$. The limiting equation then becomes

$$\text{Max } M_1(t) \leq M_2(t) \qquad t>t_0 \tag{5-3}$$

For the convex case, the third slope is needed and is represented by the following;

$$M_3(t) = \frac{F(t) - (F(t_0) - E)}{t-t_0} \tag{5-4}$$

This slope now represents the minumum value of the function over the interval in question and is thus termed Min $M_3(t)$. As described

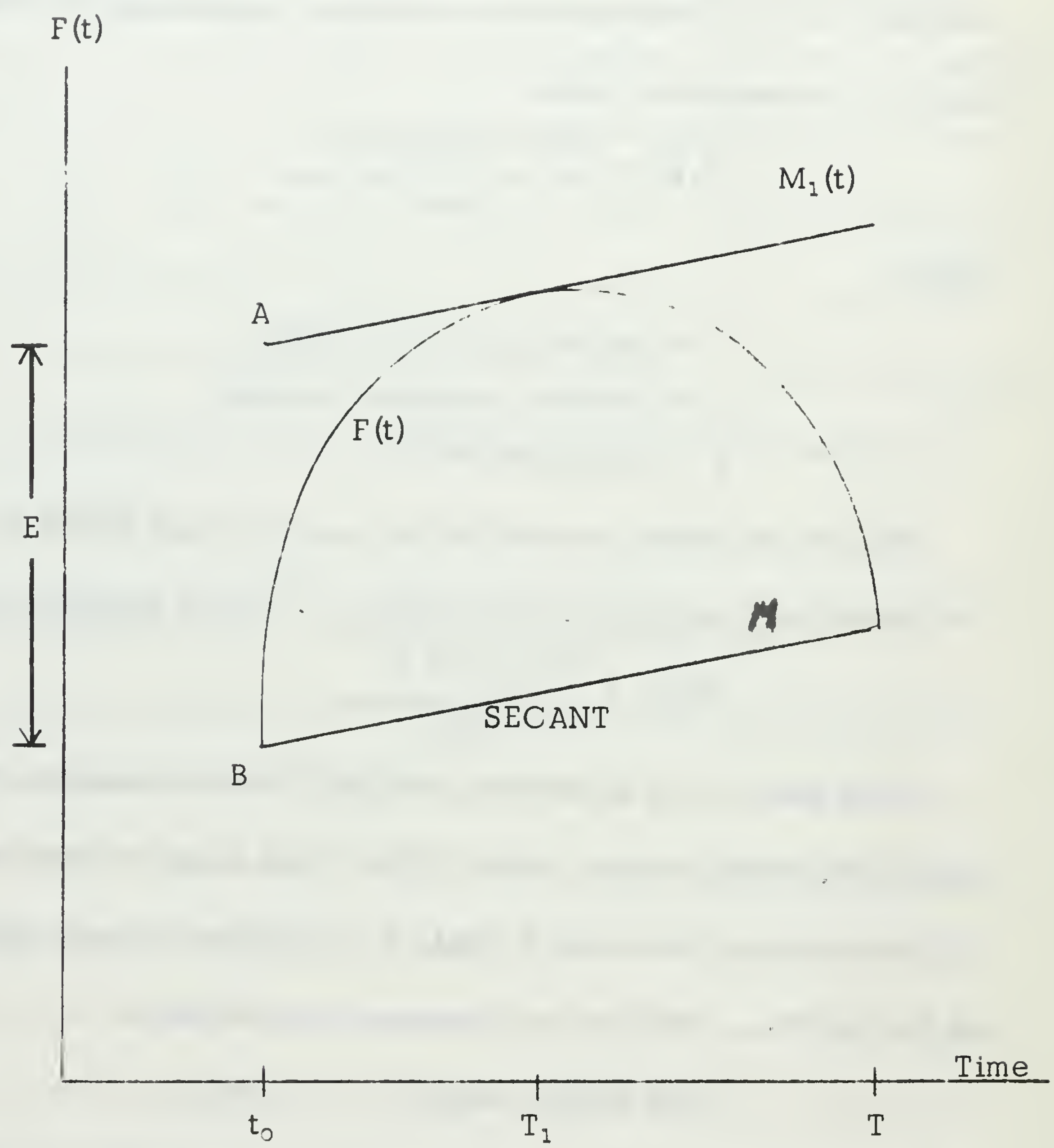


FIGURE 5.1 CONTINUOUS SECANT METHOD OF INTERPOLATION

in the reference cited for the continuous secant method, the slope of the tangent at the end of the interval is equal to $M_2(t)$ or;

$$\text{Min } M_3(t) \geq M_2(t) \qquad\qquad t>t_0$$

The maximum interval between samples is thus determined by the following relationships, depending on their direction of concavity;

$$\begin{aligned} &\text{Max } M_1(t) \leq M_2(t) \\ &\text{Min } M_3(t) \geq M_2(t) \end{aligned} \qquad\qquad t>t_0$$

All four possibilites of continuous intervals appear in Fig. 5.2. With modern hybrid computers the continuous secant method can be simulated as shown in Ref. 12.

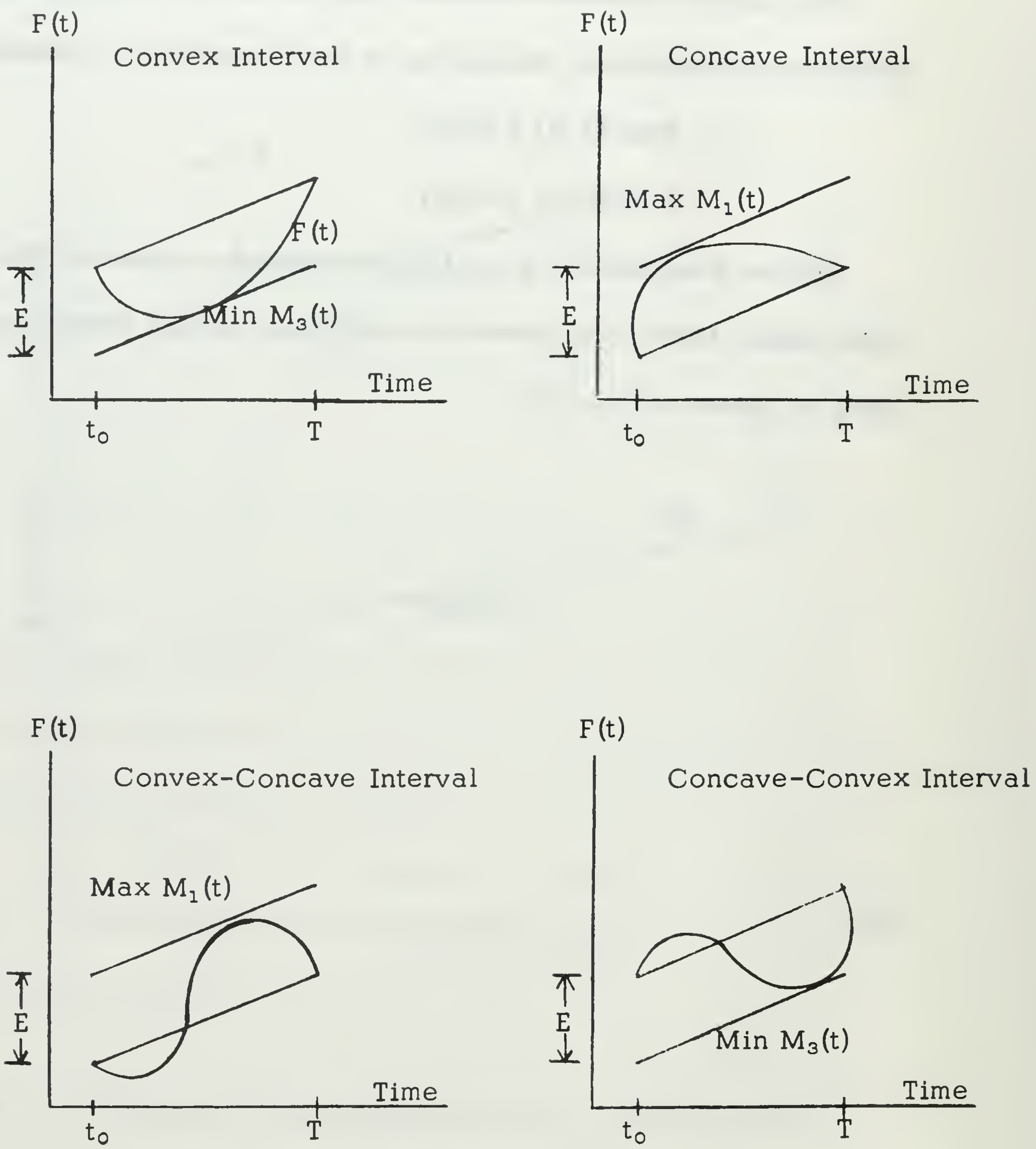


FIGURE 5.2 THE FOUR POSSIBLE RELATIONS FOR THE CONTINUOUS SECANT COMPRESSOR

VI. NOISE IN DATA COMPRESSION SYSTEMS

A. INTRODUCTION

Up to this point compression techniques have been discussed without the presence of noise. Noise is often neglected in the study of data compression because the presence of noise tends to reduce the compression ratio; and secondly, noise has a very pronounced effect on systems which use derivatives in determining redundancy.

A data compression system has a defined prescribed bandwidth which is dependent on the highest-frequency component the input signal is expected to contain. High-frequency noise beyond the sampling rate and outside of this finite bandwidth can be filtered out extremely well using conventional filtering techniques. Compression results when the system does not occupy its entire predefined bandwidth. Since one cannot filter out the noise in the prescribed bandwidth there is a problem of determining the effects of this noise on data compressors.

As was stated in Chapter III predictors tend to amplify noise and as a result they would not give an accurate representation of effects of noise on compression. In all of the interpolators discussed prior to the continuous secant method, the predefined error originated at the last transmitted sample. In the continuous secant compressor, the predefined error is placed at the point of maximum deviation of the signal from the predicted pattern. This is the main reason why the continuous secant method of compression is the most efficient of all compressors examined.

Reference 7 contains a complete analysis comparing the noise level to the number of significant samples required to reproduce the input signal to within a specified tolerance. It shows that if the RMS noise level is equal to the predefined error in the continuous secant compressor, there is an increase of only 20% in the sampling rate. The slope of the line $M_2(t)$ determines when the next sample will occur. A good measure of the effects of the noise on the compressor would be the probability distribution of the slope of the line $M_2(t)$ since it is the limiting factor in determining the optimum sampling interval.

B. CHARACTERISTICS OF THE INITIAL TEST SIGNALS

A simple sine wave was chosen for the input signal in the initial test. The sine wave has a continuously varying slope which should give good results for piecewise examination. The frequency of the sine wave is one cycle per second and the magnitude is unity.

The characteristics of the noise which was added to the sine wave are shown in Fig. 6.1. The RMS_{noise} is the root-mean-square value of the noise as compared to the input signal. The bandwidth of the noise spectrum is twenty cycles per second, centered about the origin.

C. RESULTS OF THE INITIAL TEST

The distribution of the slope was first examined for a purely concave interval represented by the first quarter of the sine wave. Various prescribed errors were used coupled with different levels of noise. The effects of the noise on the slope can be seen in Fig. 6.2a.

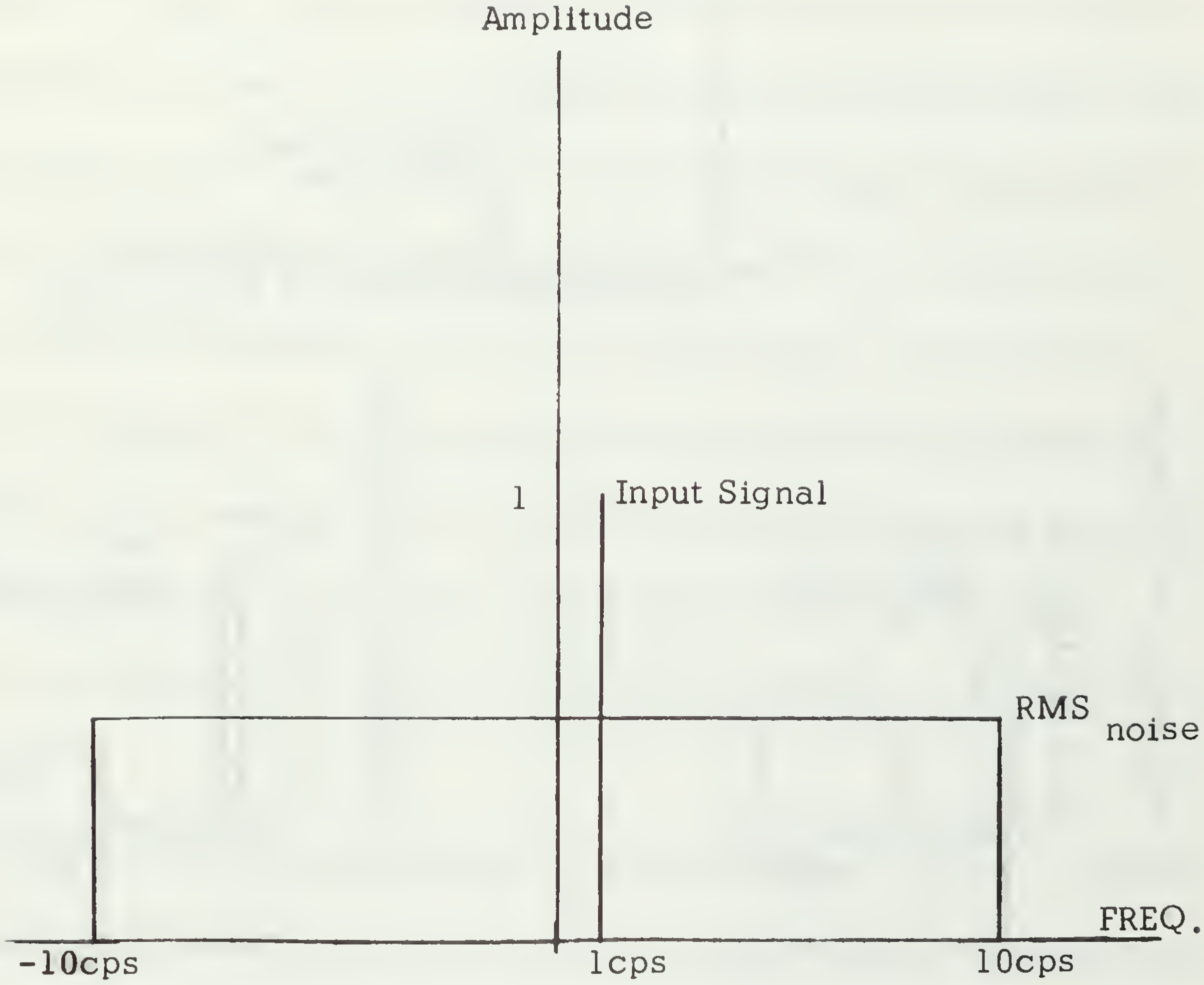


FIGURE 6.1 CHARACTERISTICS OF THE INPUT NOISE

FIRST QUARTER OF THE SINE WAVE

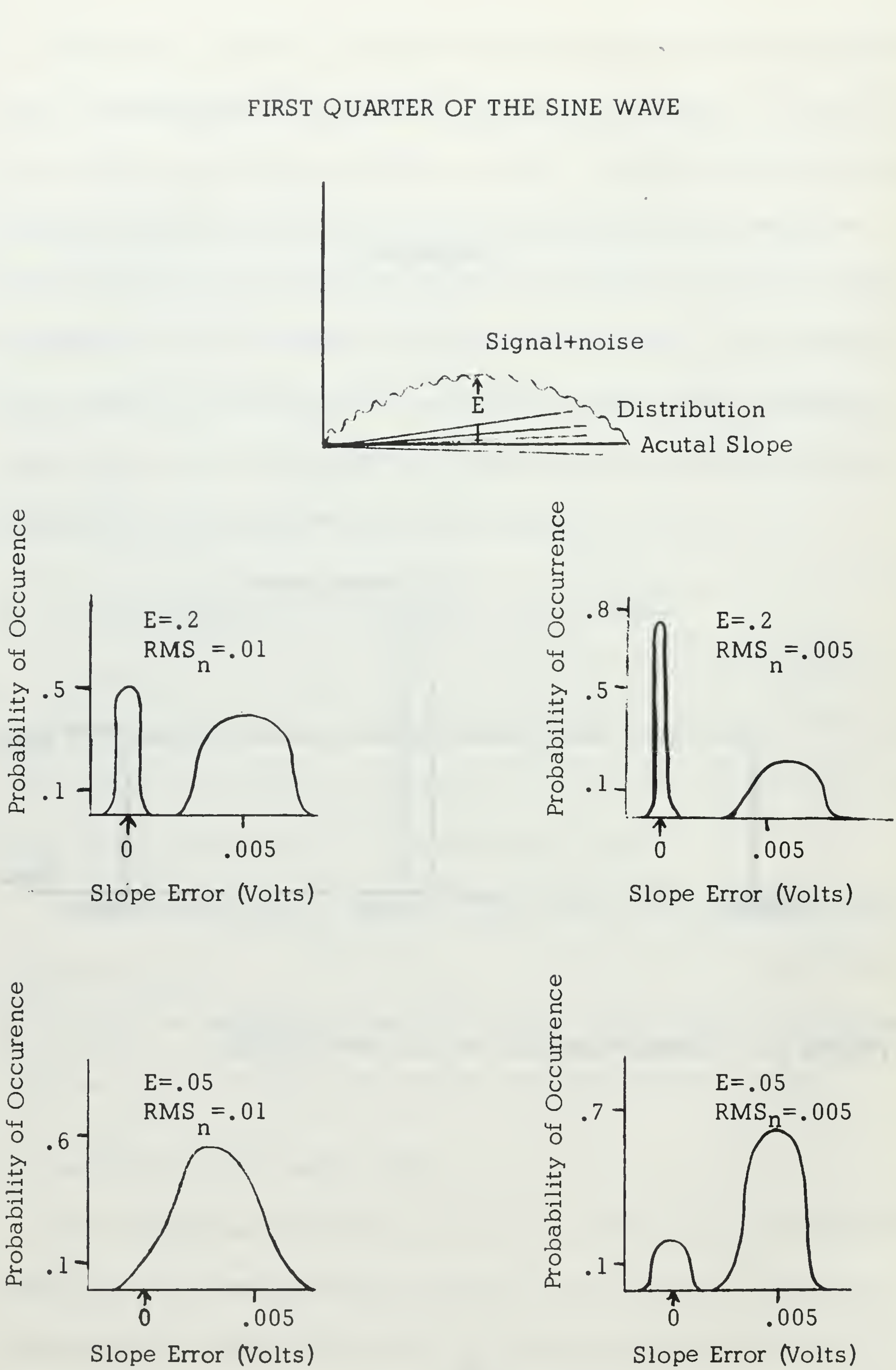


FIGURE 6.2a THE SLOPE ERROR DISTRIBUTION OF THE FIRST QUARTER OF THE SINE WAVE

At the beginning of the sine wave the slope rises very fast and as the sine wave reaches ninety degrees the slope has decreased to zero. The results of Fig. 6.2a show that the steep slope at the beginning of the sine wave had a pronounced effect on the distribution of the slope. Although for very small errors the distribution of the slope centers on the actual value of the slope with little variation observed, for significant errors the slope distribution tends to move toward a much higher value. This is due to the variation of the initial steep slope. As the noise is increased slightly, the average slope moves towards the direction of concavity until a point is reached where the distribution of the slope is completely random. This means that the input signal is completely saturated with noise and cannot be reconstructed with any degree of accuracy.

The sine wave was then shifted by one quarter of a cycle. Although the segments are the reciprocal of each other, the results were not identical. The difference in the distribution is due to the initial slope. As was shown above, the initial slope had a pronounced effect on the distribution. As shown in Fig. 6.2b, as the noise is increased the distribution shifts slightly to a more negative value. This is due to the increase in the negative slope. As the predefined error is increased more of the signal is included in the continuous interval, and the effects of the steep negative slope take hold.

The next logical step was to examine the inflection point. The results were that the distribution was nearly gaussian. If the portion of

SECOND QUARTER OF THE SINE WAVE

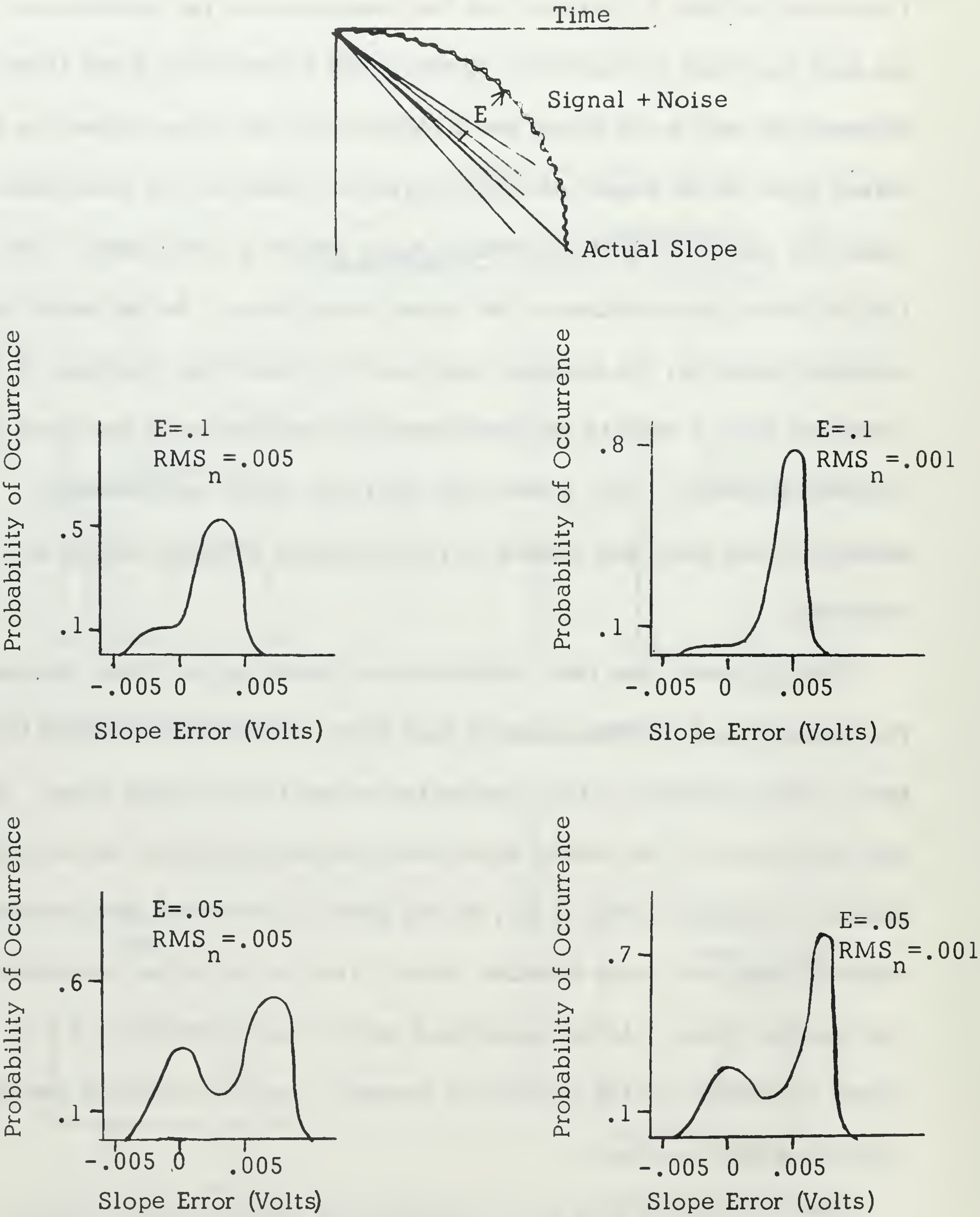


FIGURE 6.2b THE SLOPE ERROR DISTRIBUTION FOR THE SECOND QUARTER OF THE SINE WAVE

the signal is centered so that the inflection point occurs in the middle of the test signal. Of course this is not true for every inflection point on other input signals, but since the sine wave provided symmetry about the inflection point, the distribution is gaussian until the system is completely saturated by noise. By displacing the inflection point off center, the distribution is still approximately gaussian, but by adding noise the slope is offset in the direction of the maximum deviation. (See Fig. 6.2c and 6.2d)

D. EXAMINATION OF THE SLOPE IN A COMPLETELY RANDOM ENVIRONMENT

1. Characteristics of the Input Signals

In order to examine the probability distribution of the slope in a completely random environment, a random test signal was chosen. A signal composed of the sum of eight sine waves with no overlapping harmonics has the following characteristics; [Ref. 9]

a. a flat spectral density

b. a probability distribution which is identical with a gaussian distribution

The frequencies of the new test signal are as follows:

$$S_{noise}(t) = \sum_{i=1}^{i=8} \text{Sin}\,(W_i t)$$

W_1 = .1122 radians/sec

W_2 = .1995 radians/sec

W_3 = .2860 radians/sec

W_4 = .3940 radians/sec

W_5 = .4910 radians/sec

INFLECTION POINT

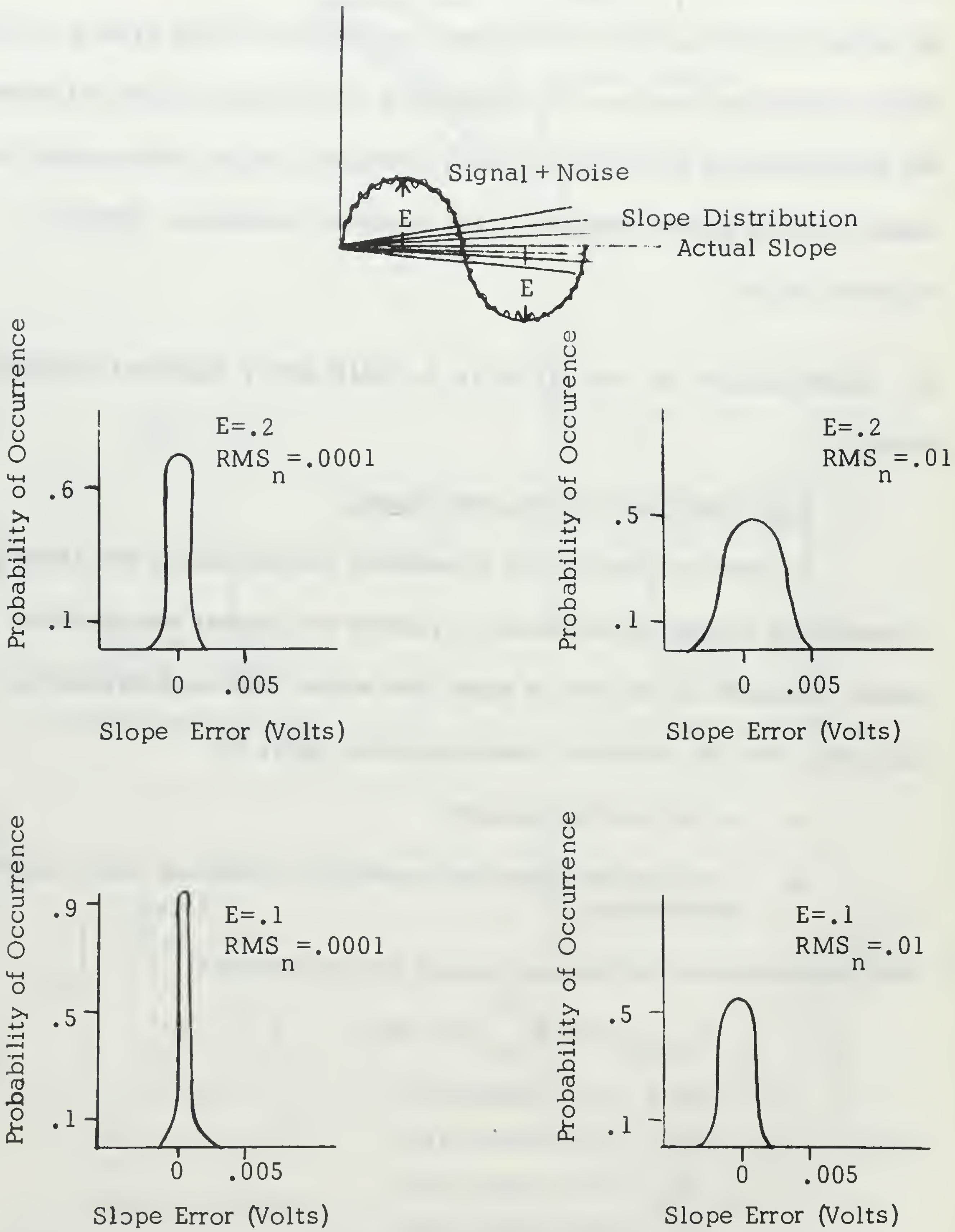


FIGURE 6.2c THE SLOPE ERROR DISTRIBUTION FOR THE INFLECTION

OFFSET INFLECTION POINT

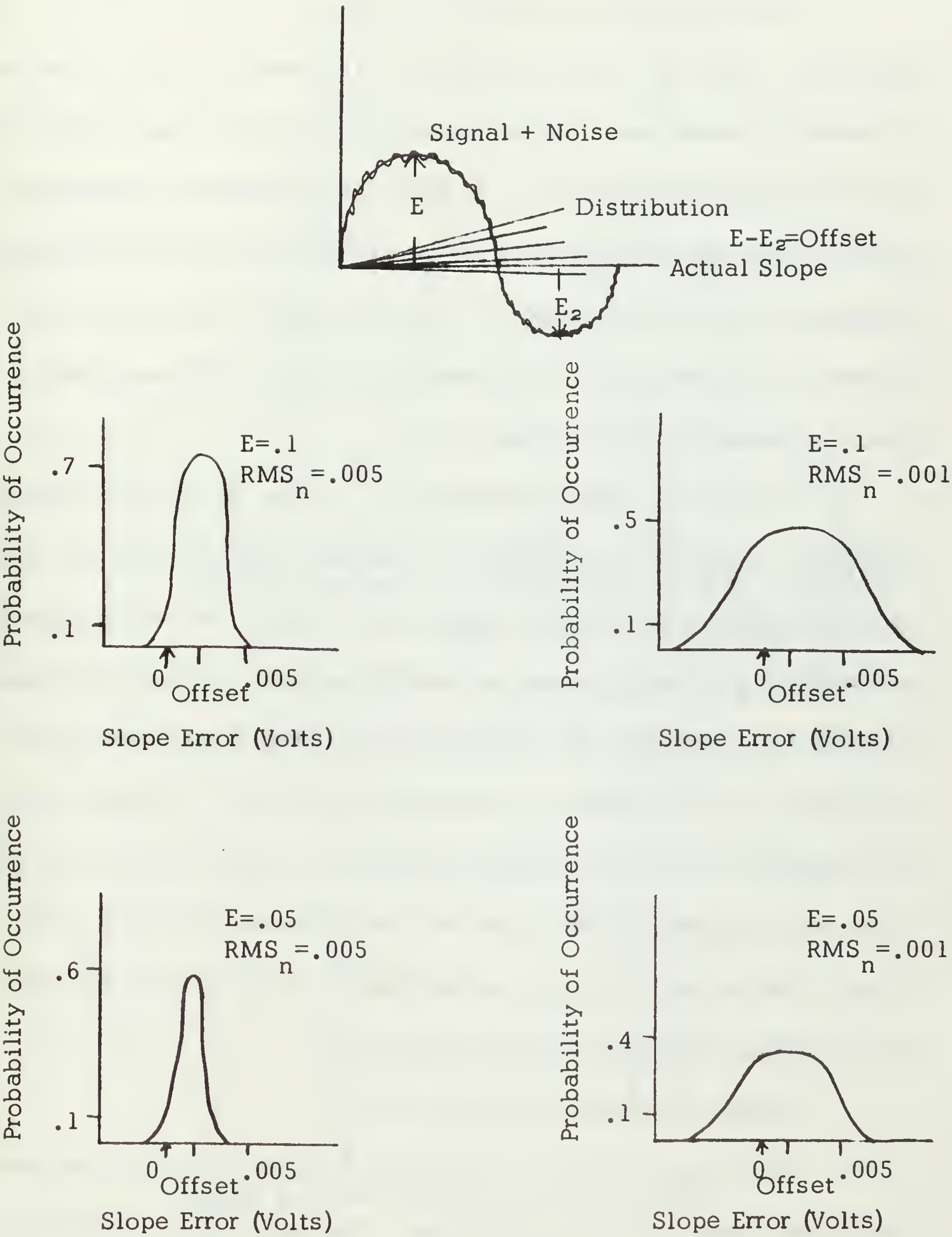


FIGURE 6.2d THE SLOPE ERROR DISTRIBUTION FOR THE OFFSET INFLECTION POINT

W_6 = .6100 radians/sec

W_7 = .7320

W_8 = .8250

The noise has the same characteristics as used in the previous examples. Since the noise bandwidth is still twenty cycles per second, it covers the entire range of the carrier frequencies and has ample width to cover significant harmonics. In order to approximate a continuous signal on the SDS 9300 digital computer a sampling rate of five hundred cycles per second was chosen. A fourier analysis of the input signal showed that the signal had a flat spectral density. The amplitude of the input signal is shown in Fig. 6.3.

To insure a random starting point another gaussian distribution was used. As was seen in the initial test, the starting point can have a pronounced effect on the distribution of the slope. The distribution of the starting point had a gaussian distribution with a standard deviation equal to five seconds. The positive half of the gaussian distribution was used in order to insure a positive starting point. The input signal was examined from zero to fourteen seconds. At each random point the slope was computed for the signal without noise and the signal plus noise. The actual slope was compared to the slope of the signal plus noise and the resultant error was determined.

2. Results of the Random Signal Test

The results of the error distribution show that the slope error had a fairly good gaussian distribution. The one exception was that for small predefined errors the probability distribution seemed to shift

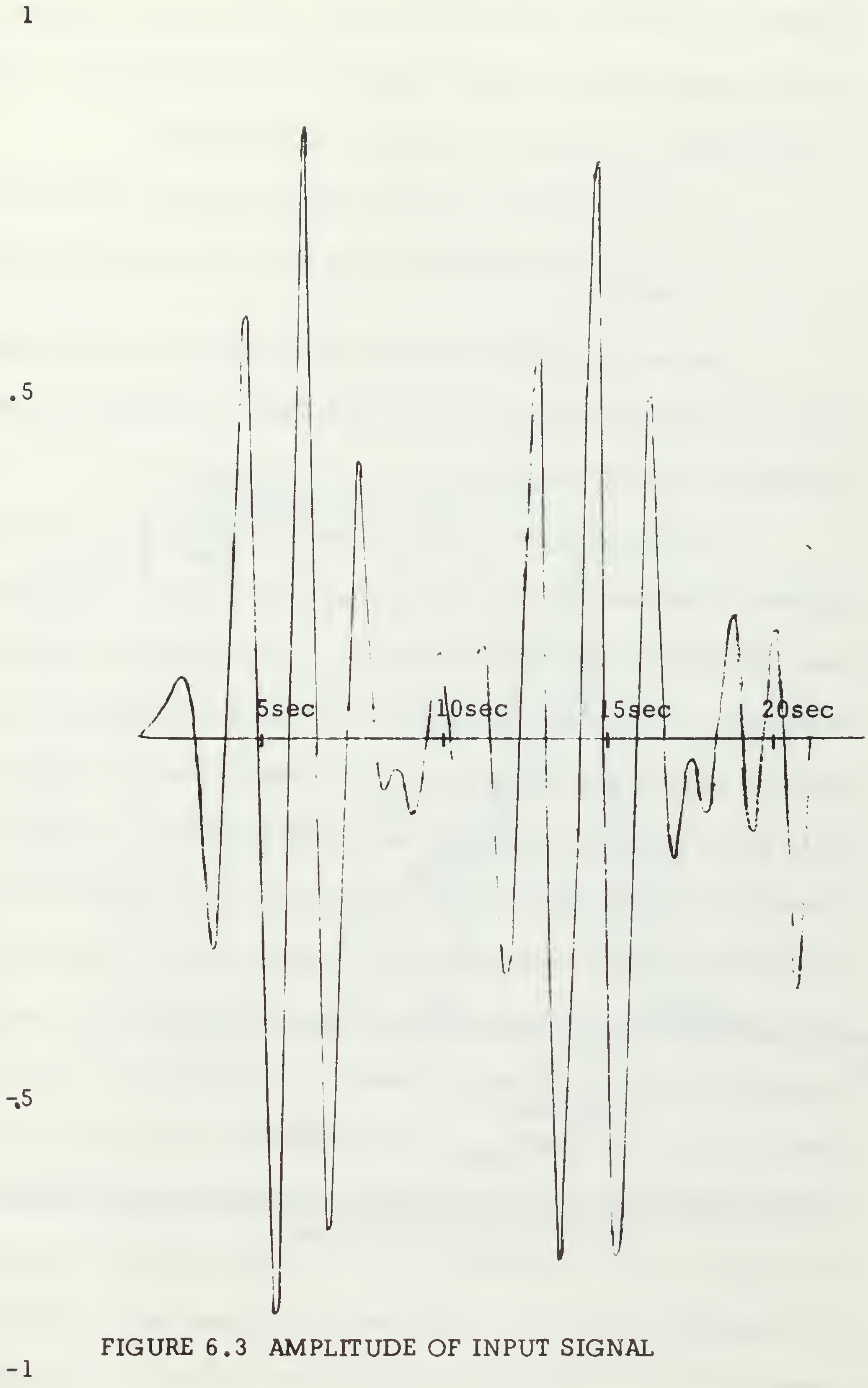


FIGURE 6.3 AMPLITUDE OF INPUT SIGNAL

slightly to a smaller slope for high noise, and for high predefined errors, the distribution seemed to shift slightly to the higher slopes. There is no justification for this result except for the following:

a. Only a relatively small number of samples were examined

b. The random generator might have been biased for certain values

One can conclude that the overall distribution of the slope error is generally gaussian in form. (See Figs. 6.4-6.4e for a detailed analysis of each individual run)

The following set of graphs (Figs. 6.5a-6.5e) present a slightly different interpretation of the slope error distribution. The horizontal axis represents the percentage of RMS_{noise} as compared to the predefined error, and the vertical axis represents the probability that the slope will fall between the specified error limitations labeled on each curve. As is shown in the graphs, when the predefined error is increased the slope of each of the curves drops significantly as compared to the same curve with a lower predefined error. As an example, consider a probability of 60% that the slope error will be less than $\pm$ .0001, and that the system has a predefined error tolerance of .01; from Fig. 6.5b it is found that the ratio of the RMS_{noise} to the predefined error can be as high as 8.5%, or the RMS_{noise} can be .0085. For the same probability with a predefined error of.1, the RMS_{noise} can now be .06 which is an increase in predefined error of ten. These results indicate that by increasing the predefined error and keeping the noise level constant, the probability of

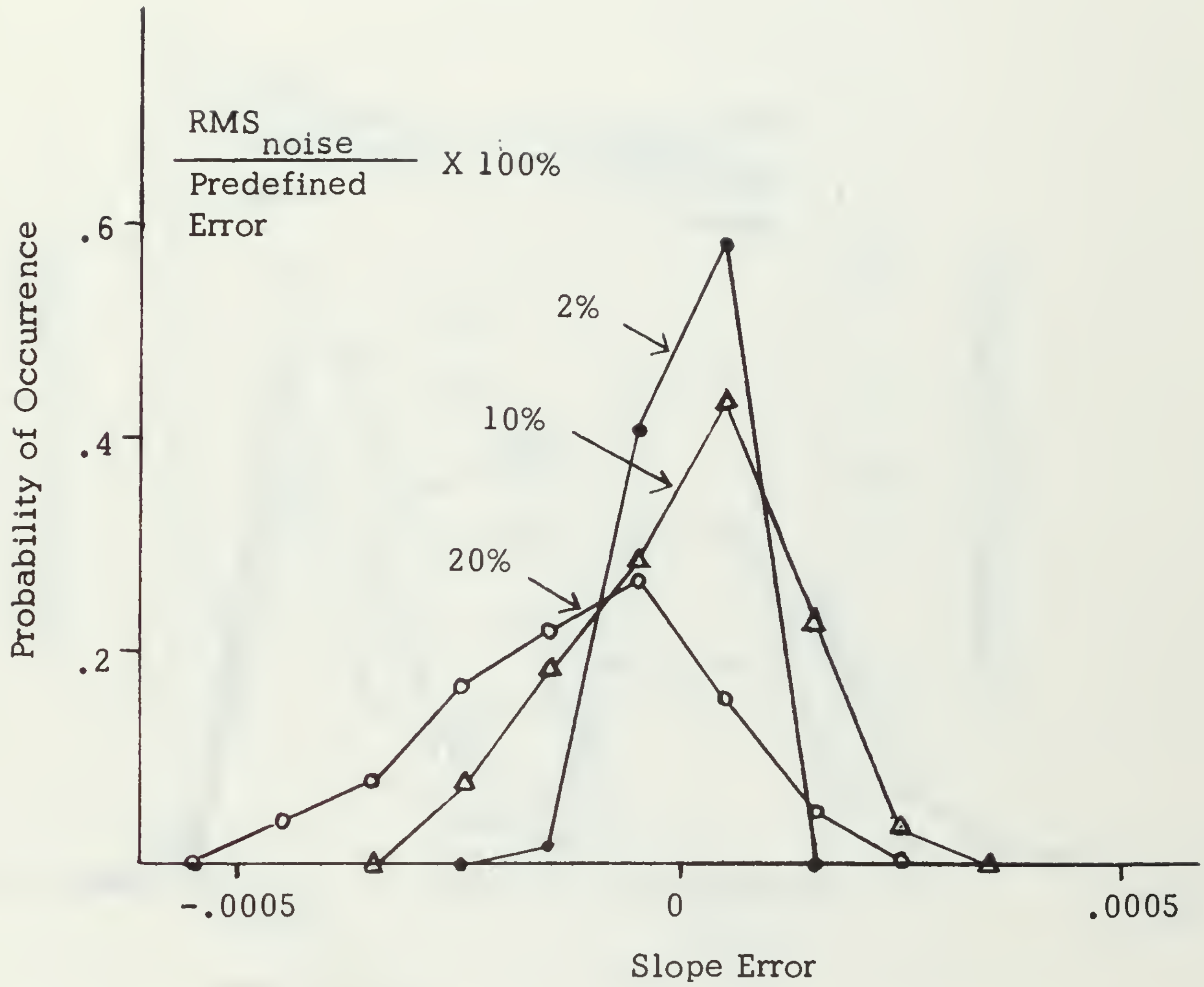


FIGURE 6.4a PROBABILITY DISTRIBUTION OF SLOPE ERROR FOR A PREDEFINED ERROR OF .005

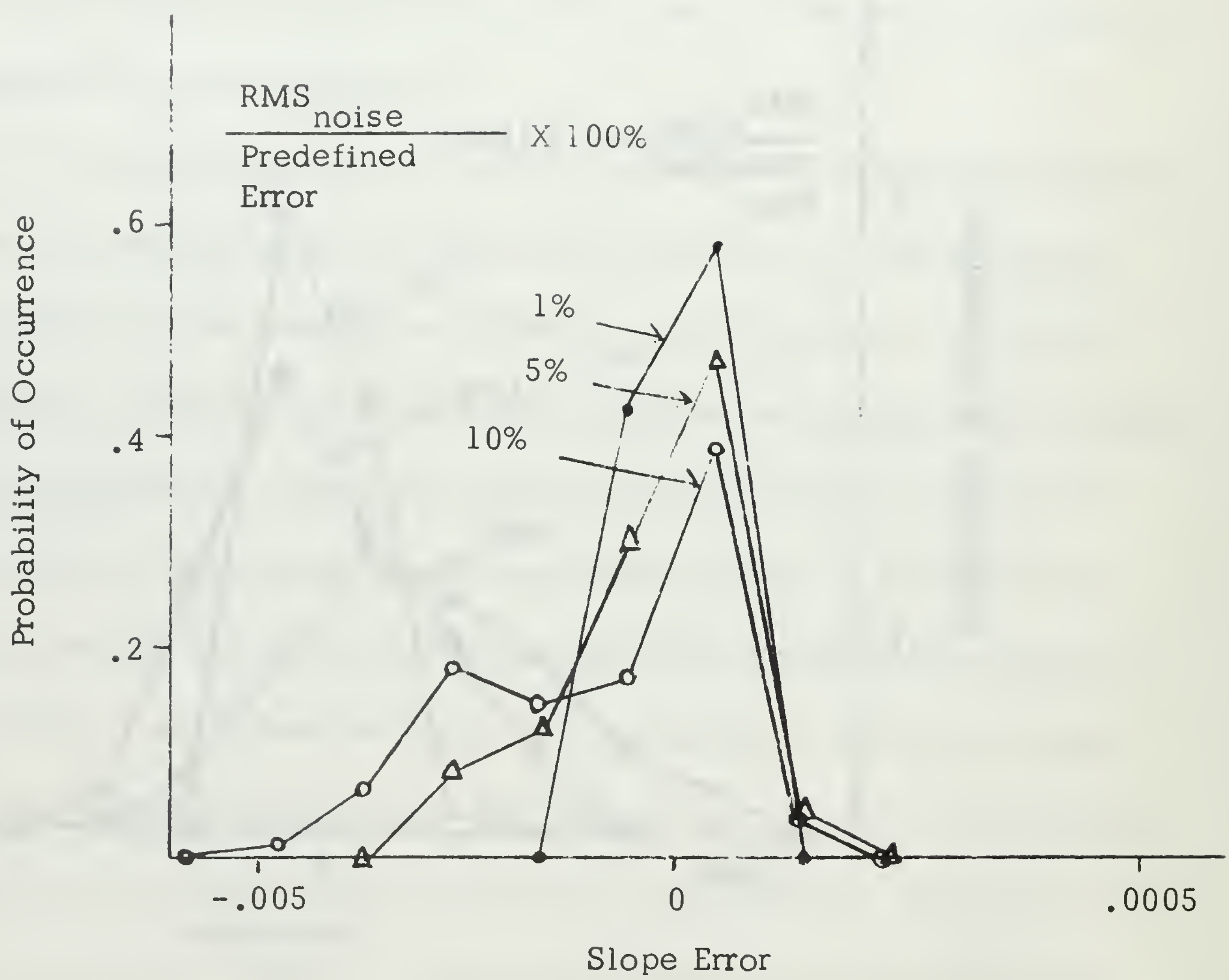


FIGURE 6.4b PROBABILITY DISTRIBUTION OF SLOPE ERROR FOR A PREDEFINED ERROR OF .01

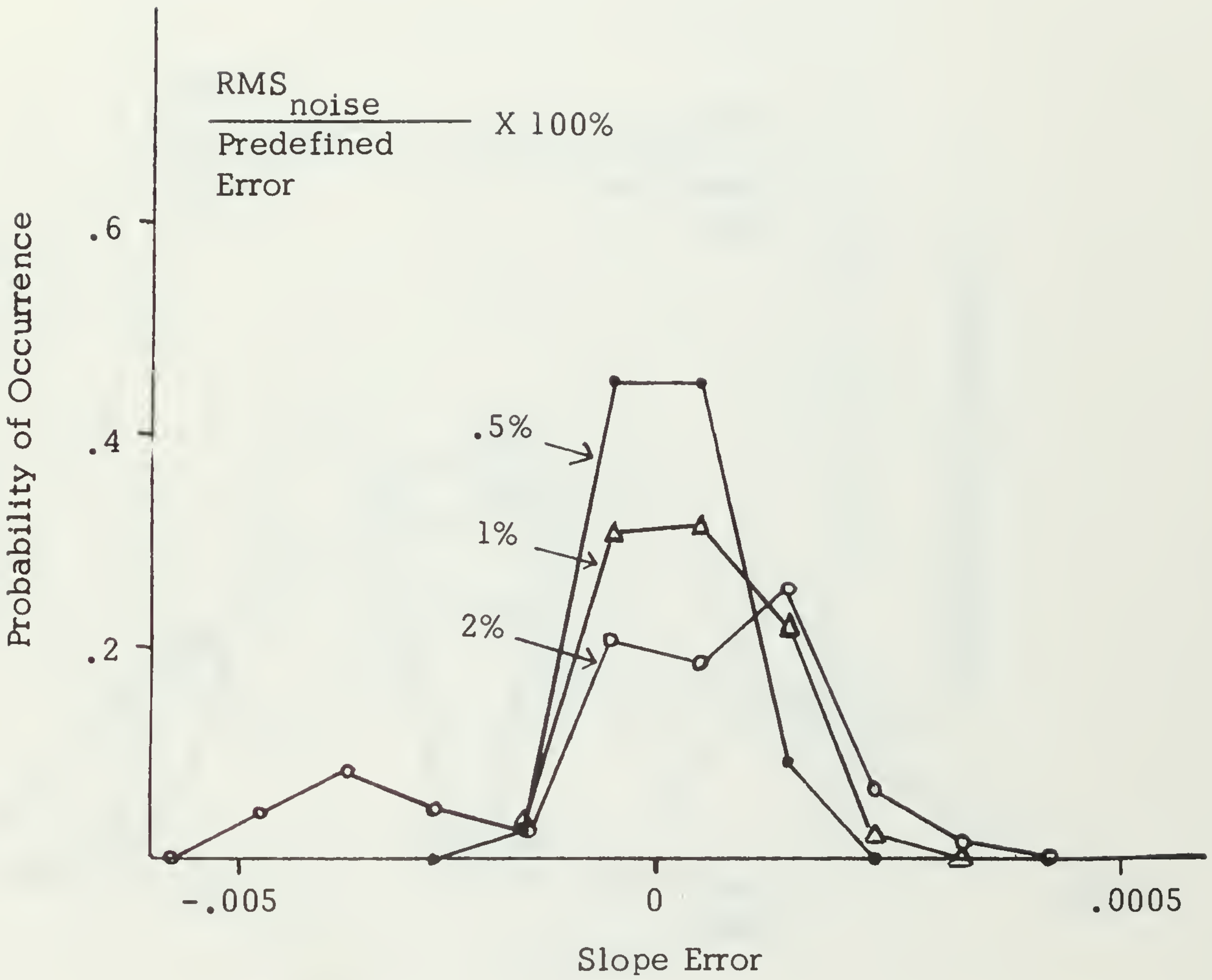


FIGURE 6.4c PROBABILITY OF SLOPE ERROR FOR A PREDEFINED ERROR OF .05

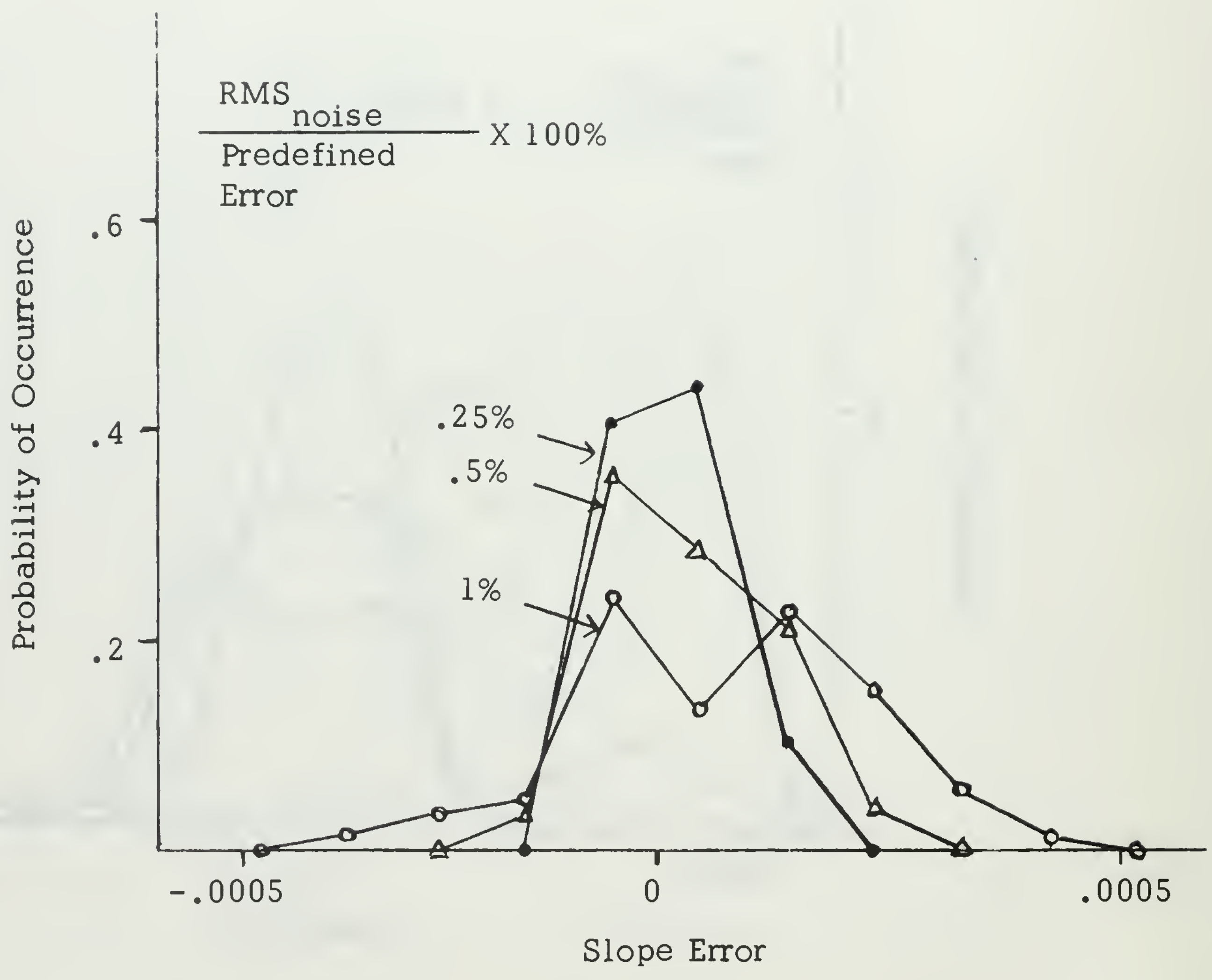


FIGURE 6.4d PROBABILITY DISTRIBUTION OF SLOPE ERROR FOR A PREDEFINED ERROR OF .01

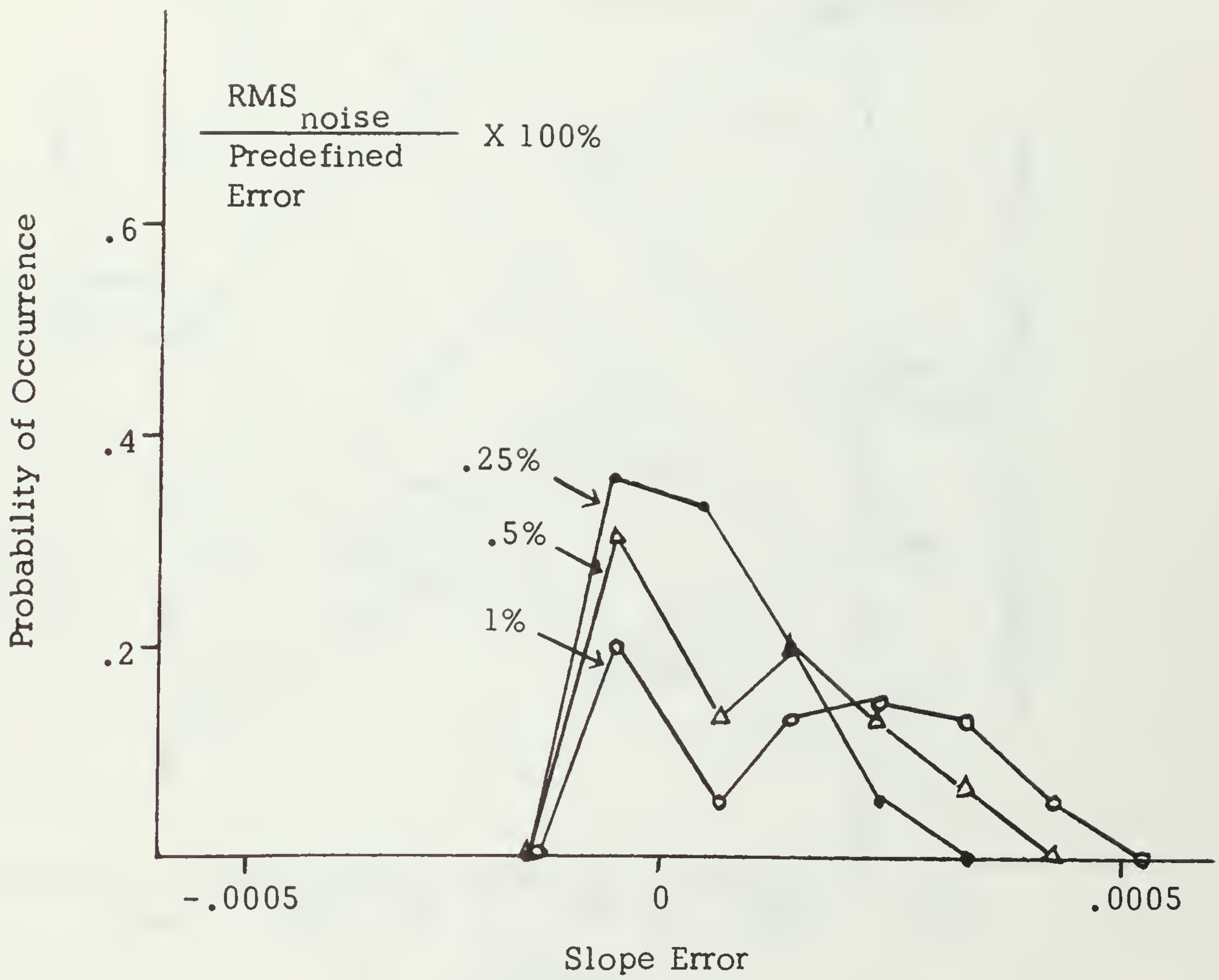


FIGURE 6.4e PROBABILITY DISTRIBUTION OF SLOPE ERROR FOR A PREDEFINED ERROR OF .2

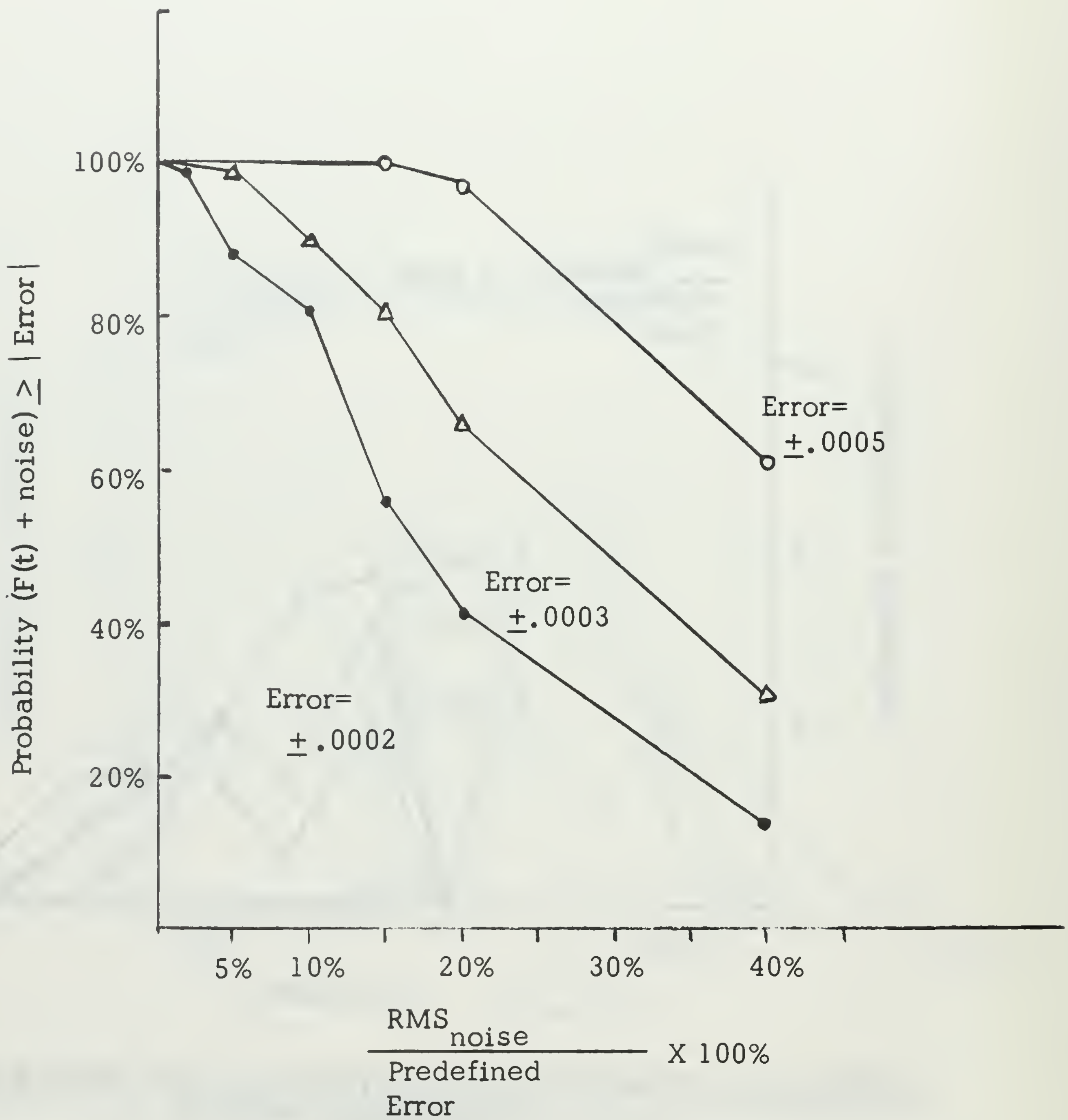


FIGURE 6.5a PROBABILITY DISTRIBUTION OF SIGNAL AND NOISE FOR A PREDEFINED ERROR OF .005

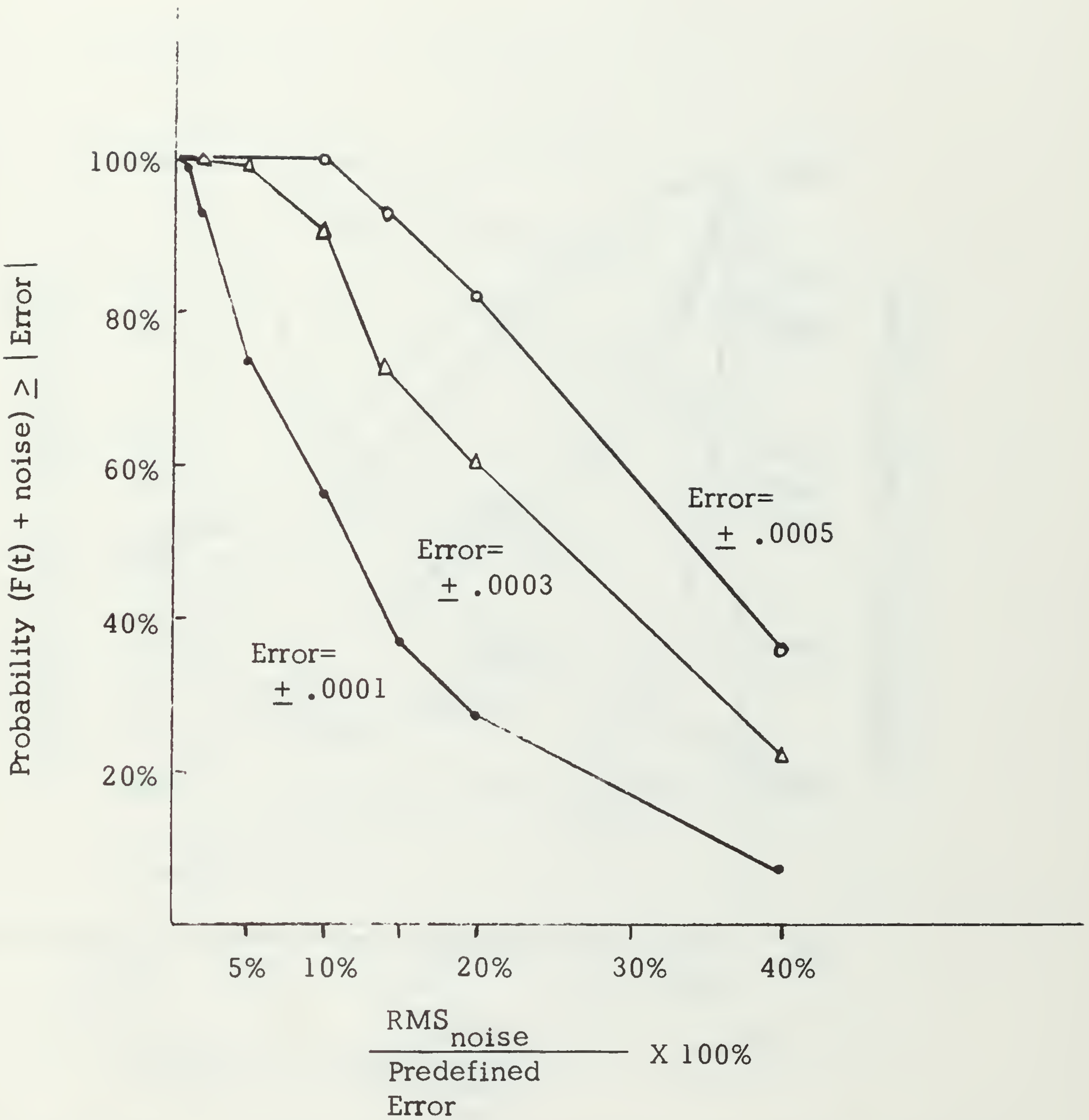
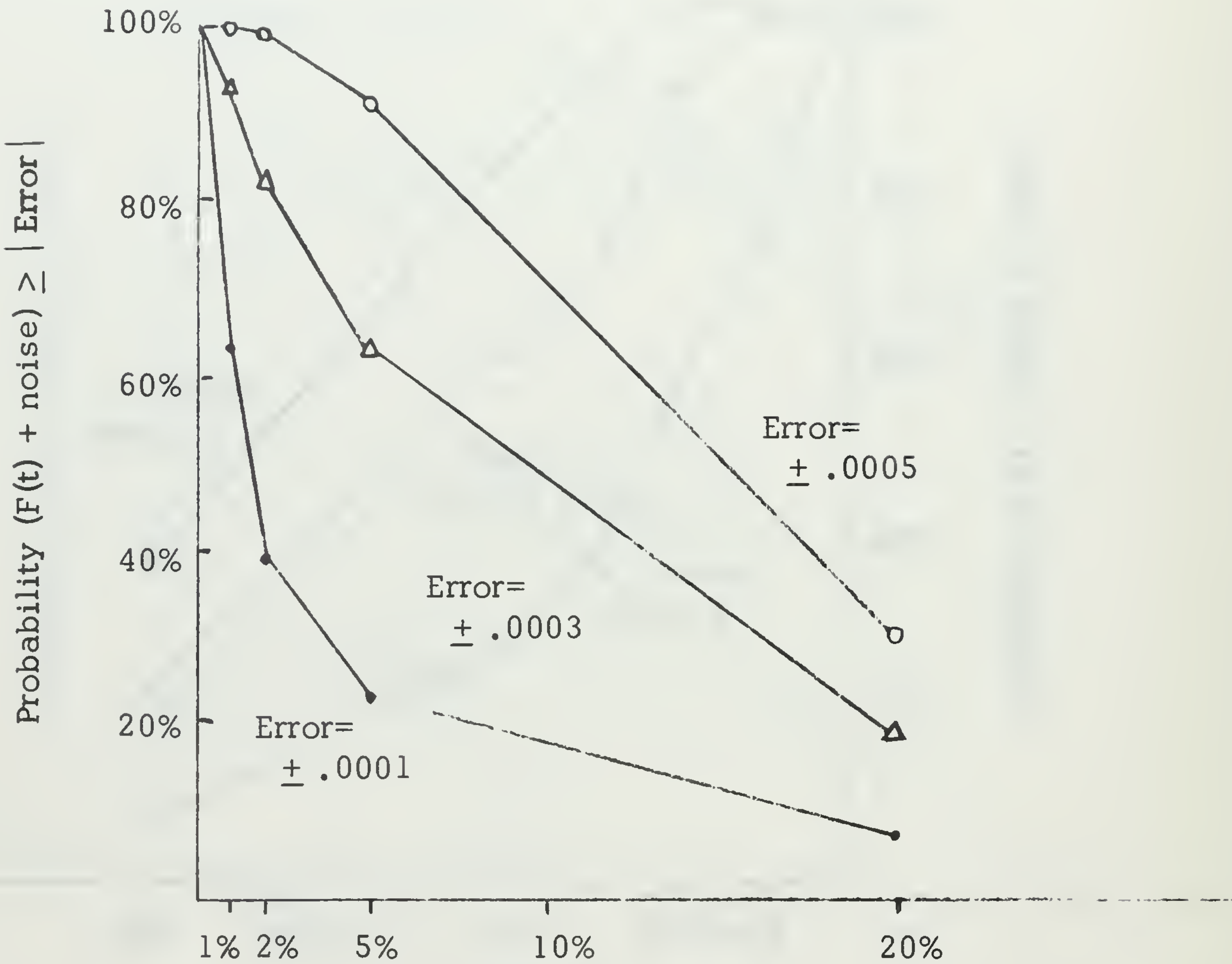


FIGURE 6.5b PROBABILITY DISTRIBUTION OF SIGNAL AND NOISE FOR A PREDEFINED ERROR OF .01



$$\frac{RMS_{noise}}{\text{Predefined Error}} \times 100\%$$

FIGURE 6.5c PROBABILITY DISTRIBUTION OF SIGNAL AND NOISE FOR A PREDEFINED ERROR OF .05

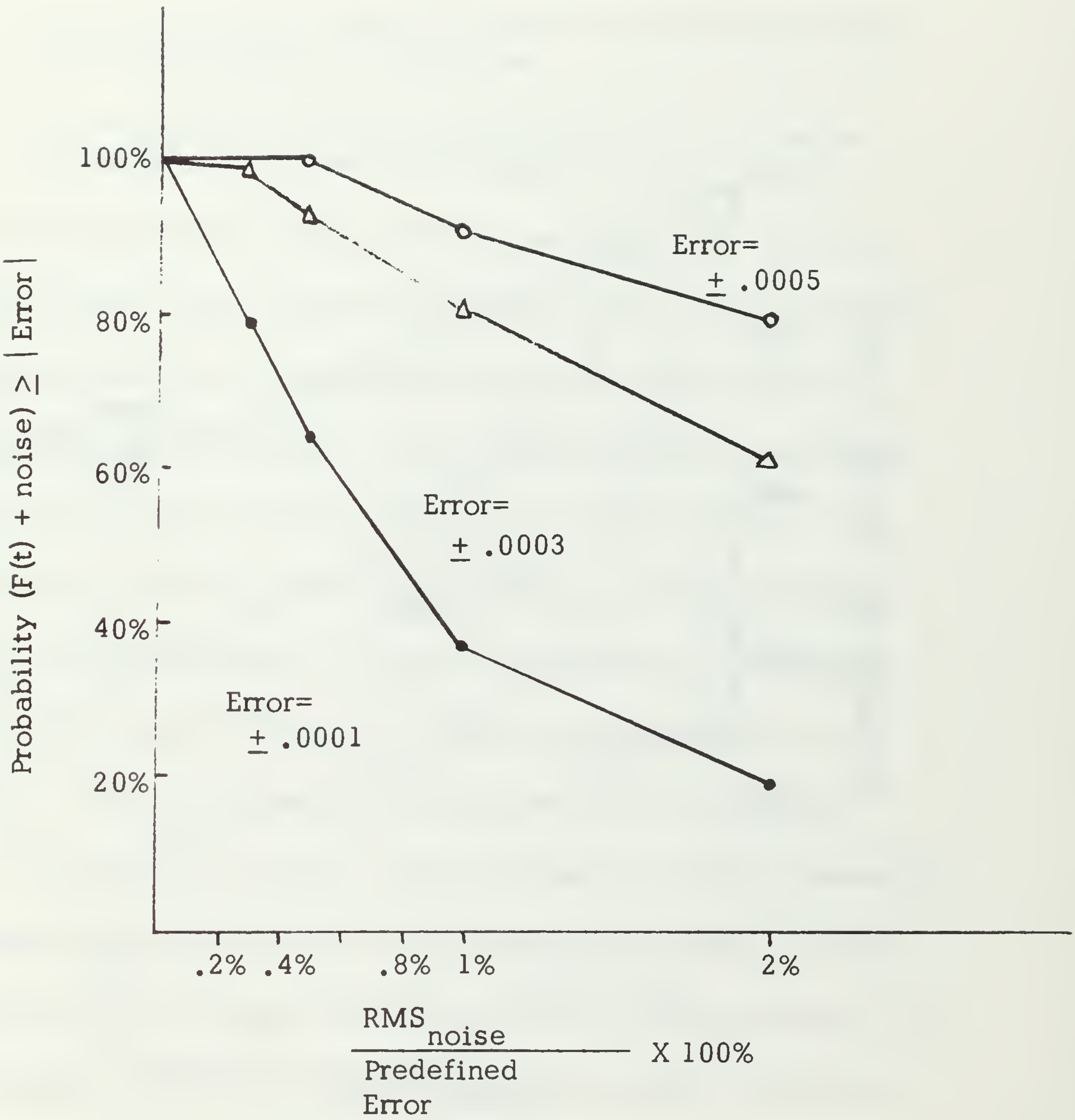


FIGURE 6.5d PROBABILITY DISTRIBUTION OF SIGNAL AND NOISE FOR A PREDEFINED ERROR OF .1

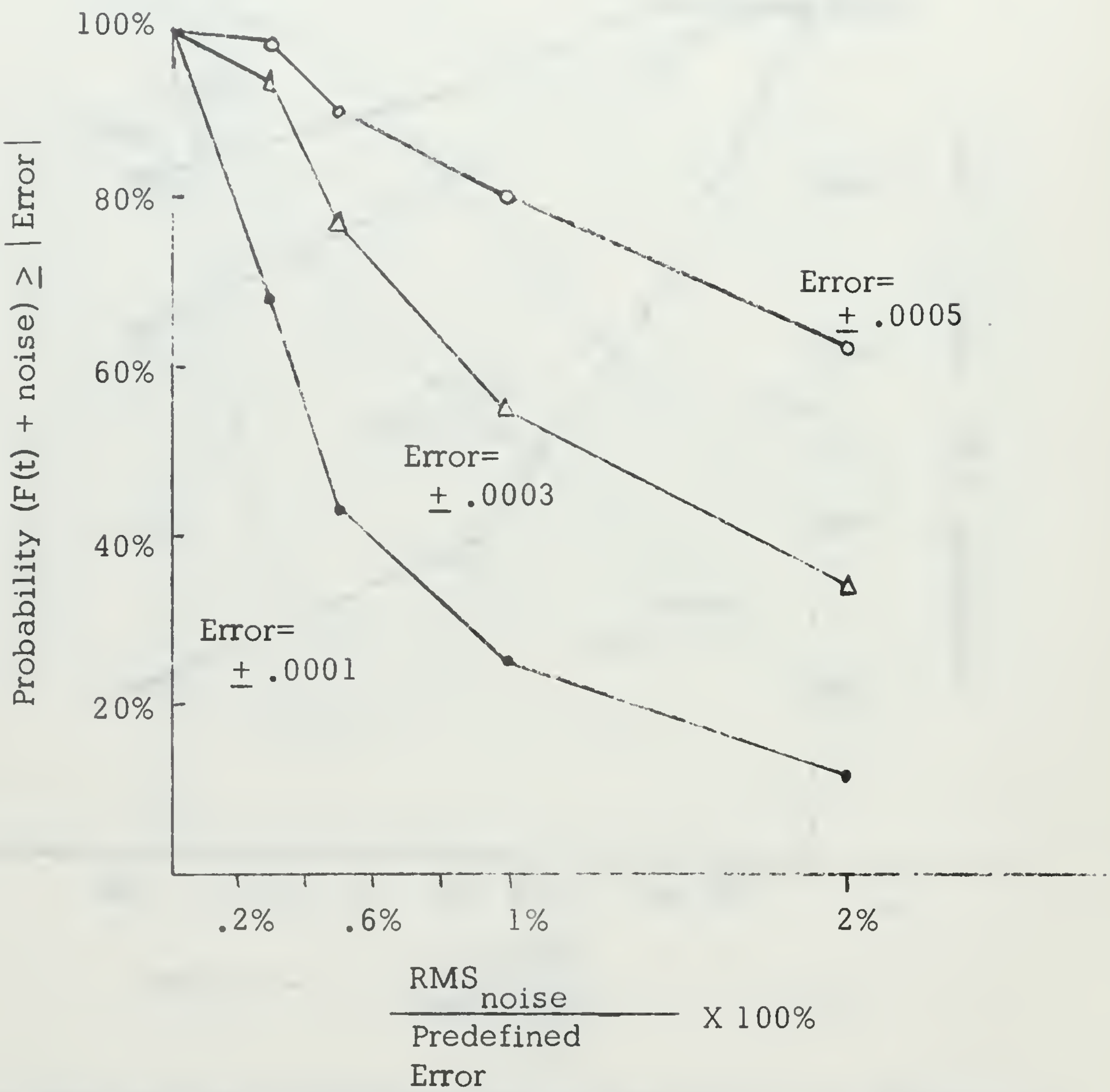


FIGURE 6.5e PROBABILITY DISTRIBUTION OF SIGNAL AND NOISE FOR A PREDEFINED ERROR OF .2

the slope being within certain limits increases and a more efficient data compressor results. There is another factor that must be considered to fully comprehend the meaning of the last statement. Increasing the predefined error causes the following:

a. The time between samples will increase (provided a fairly random input signal is used); and

b. The reconstruction of the original signal will not be as accurate as the system may require.

Although the samples may be fairly accurate, the information that was considered redundant in this system might actually be of value. Suppose that this data compressor was operating in fairly high noise, and this system was monitoring a heartbeat. The predefined error is high due to the high noise level, but the system still picks up the heartbeat. The only problem is that all the heart murmurs and small echoes would go undetected. As another example, consider the temperature in a spacecraft which might be oscillating at a very low amplitude below the prescribed error. Although the system would give better results for a higher value of noise, the presence of this oscillation might go undetected and it could be damaging to the spacecraft. Therefore, there is a trade off, or an optimum point where the predefined error will be operating, low enough to be sensitive to significant changes in the system, and high enough to keep the noise from interfering significantly with system efficiency.

E. PERCENTAGE OF SLOPE ERROR DISTRIBUTION

In the past examples, the slope error had no relation to the true slope. In order to clarify the distribution of the slope as compared to the actual value of the slope, and not to the slope error, a second set of tests was applied. These tests differed in that the slope error was divided by the actual slope to determine the percentage error.

The results of these tests gave much more accurate data than the previous tests. These graphs (Figs. 6.6a to 6.6f) do not have any significant shift in the distribution; they are fairly gaussian. The only reason for minor variations from a true gaussian pattern is that only one hundred samples were chosen at random, instead of the large numbers required to give a good gaussian pattern. Since these graphs are fairly similar in form, a correlation was discovered between each set of curves. (The results that were used in determining the curves are shown in Table I) The results show that there is actually only one set of gaussian curves. By doubling the predefined error and increasing the RMS noise-to-predefined-error ratio (Err/E) by a factor of two, the probability distributions were almost identical. This means that by doubling the predefined error, the noise can be increased by a factor of four and the same gaussian distribution will occur. This agrees with the previous test result, that the greater the predefined error, the noisier the environment can be. The reason for this is fairly obvious. As was stated earlier in the chapter, the higher the predefined error the greater the time between samples. The more signal to be examined, the higher the probability that the noise will

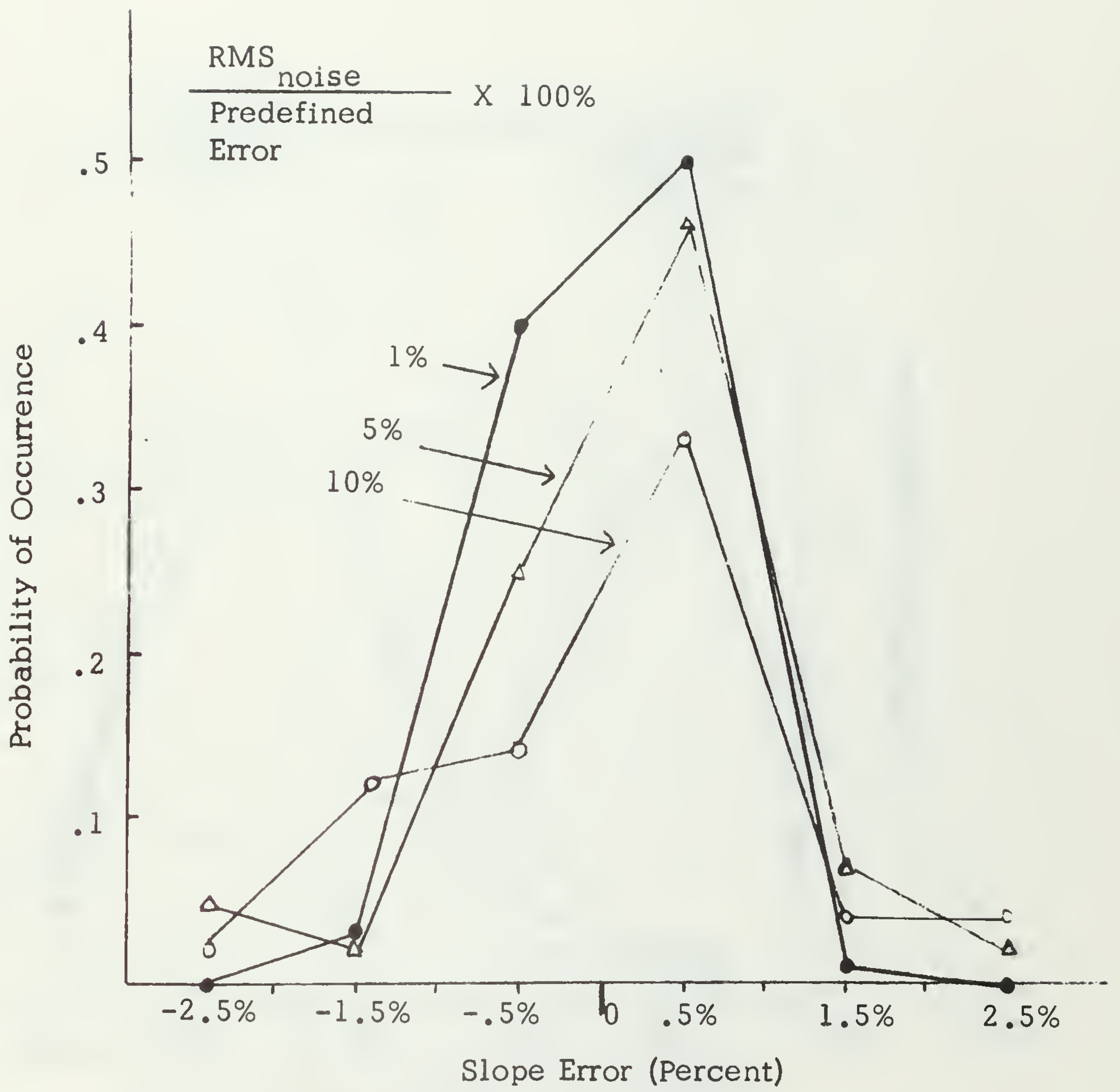


FIGURE 6.6a PROBABILITY DISTRIBUTION OF SLOPE ERROR FOR A PREDEFINED ERROR OF .001

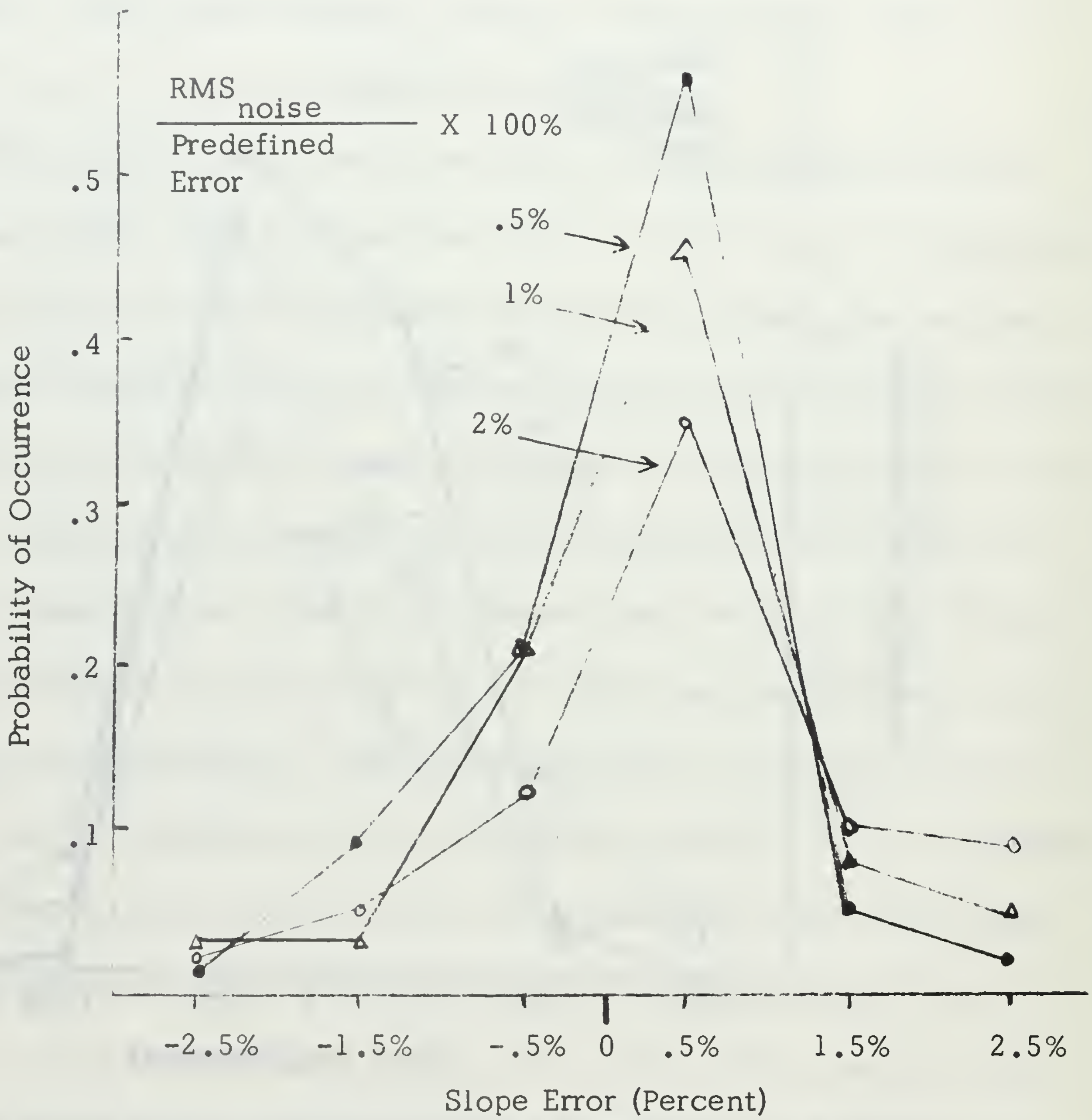


FIGURE 6.6b PROBABILITY DISTRIBUTION OF SLOPE ERROR FOR A PREDEFINED ERROR OF .005

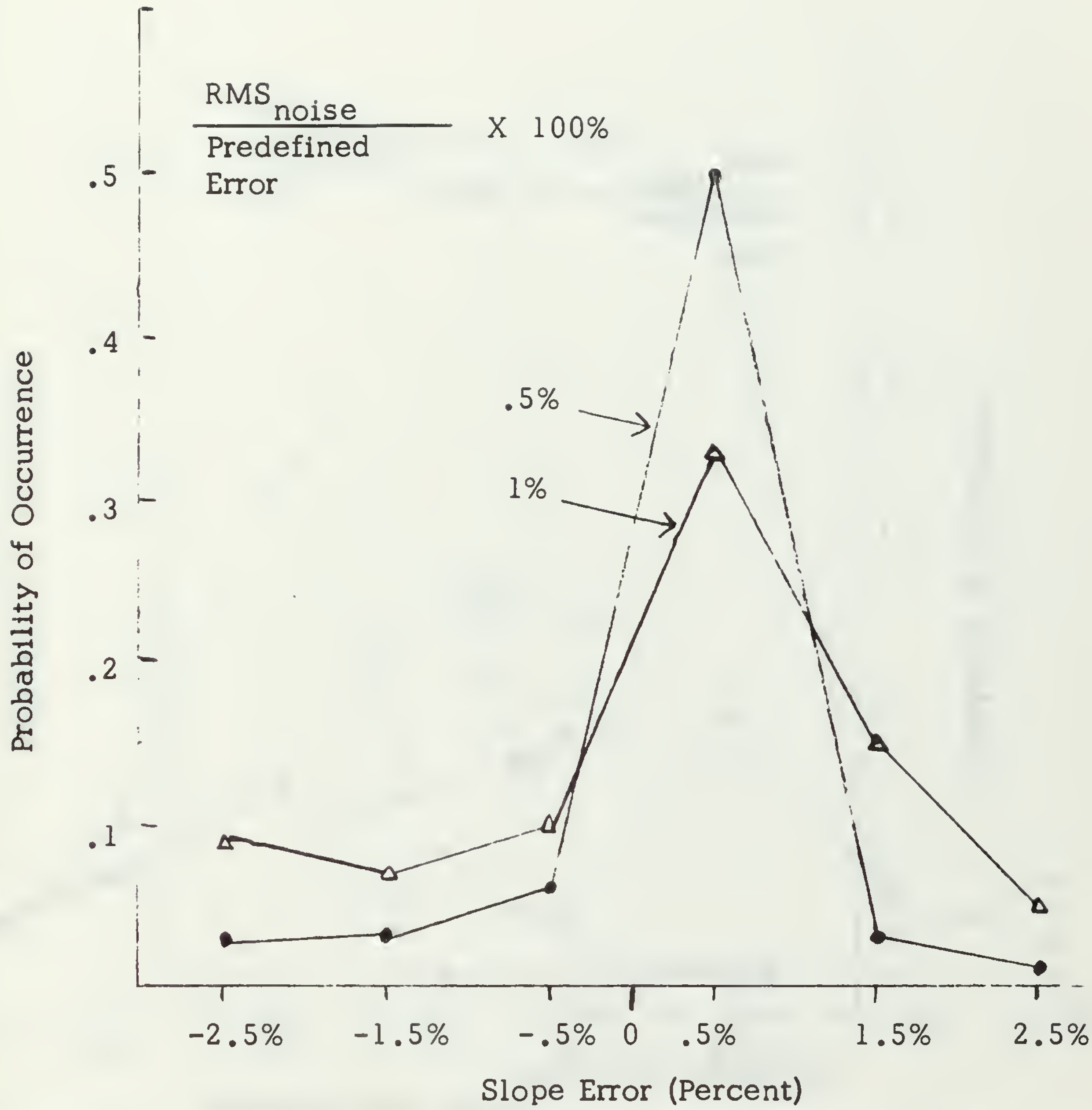


FIGURE 6.6c PROBABILITY DISTRIBUTION OF SLOPE ERROR FOR A PREDEFINED ERROR OF .01

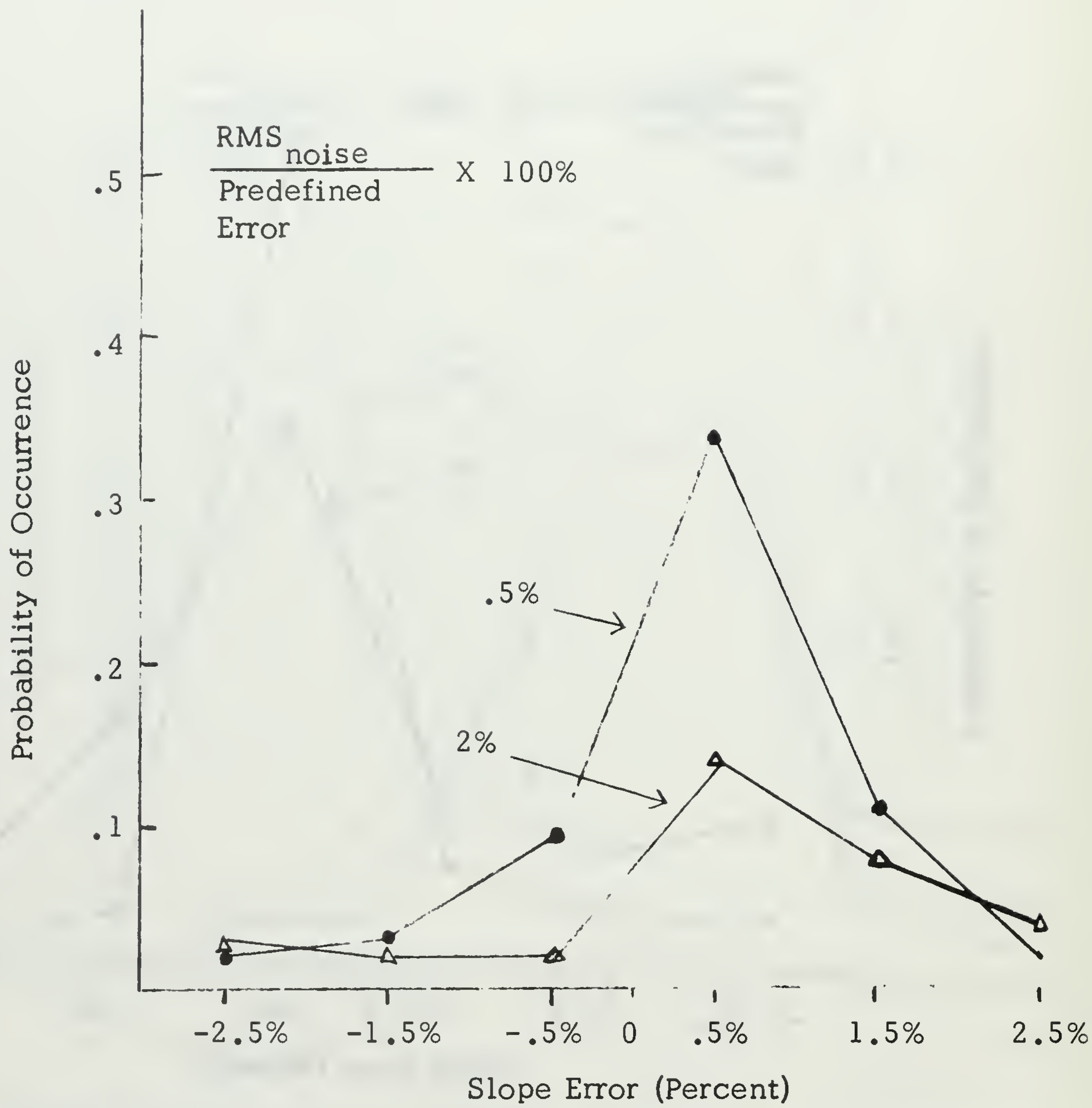


FIGURE 6.6d PROBABILITY DISTRIBUTION OF SLOPE ERROR FOR A PREDEFINED ERROR OF .02

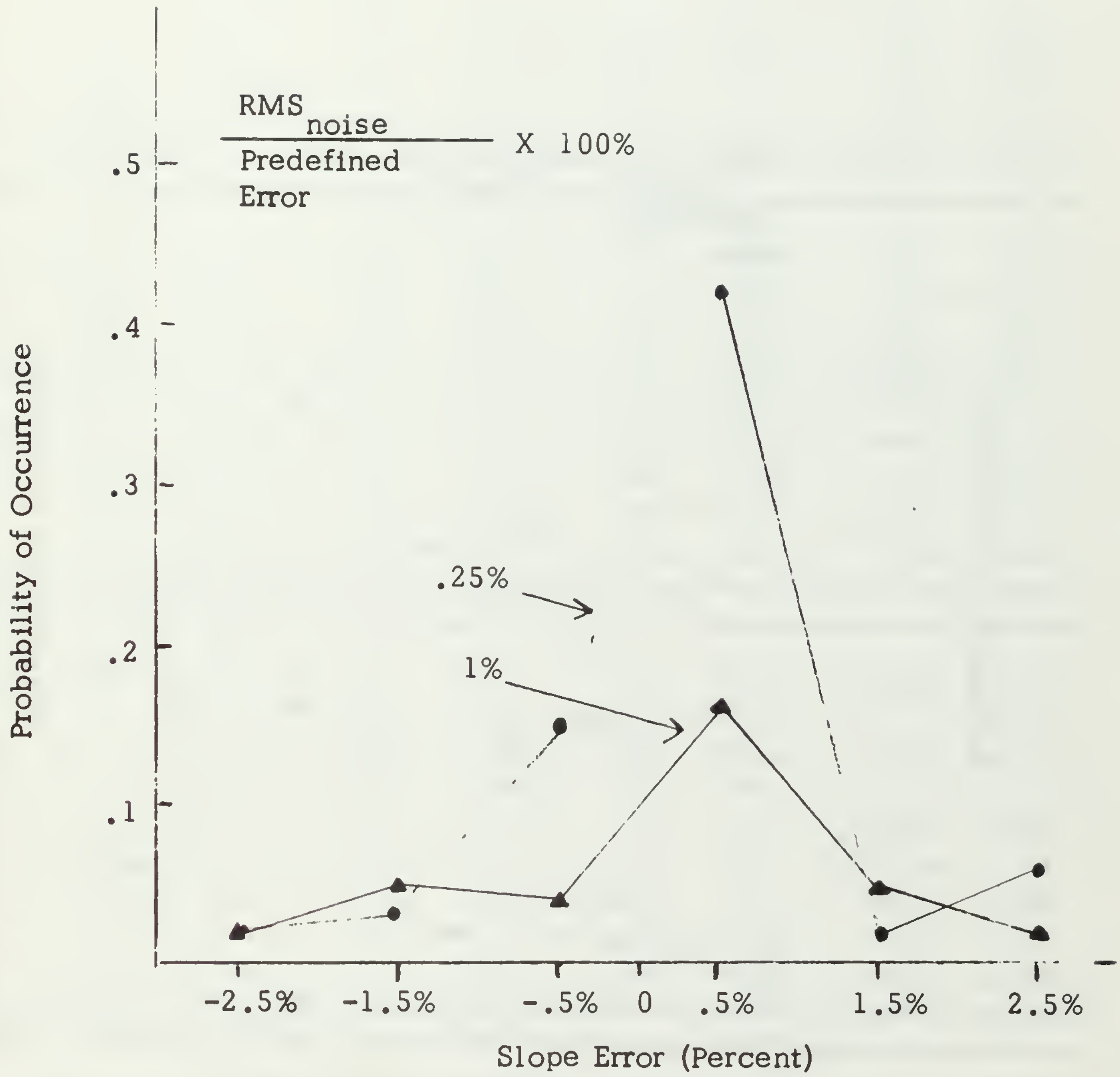


FIGURE 6.6e PROBABILITY DISTRIBUTION OF SLOPE ERROR FOR A PREDEFINED ERROR OF .05

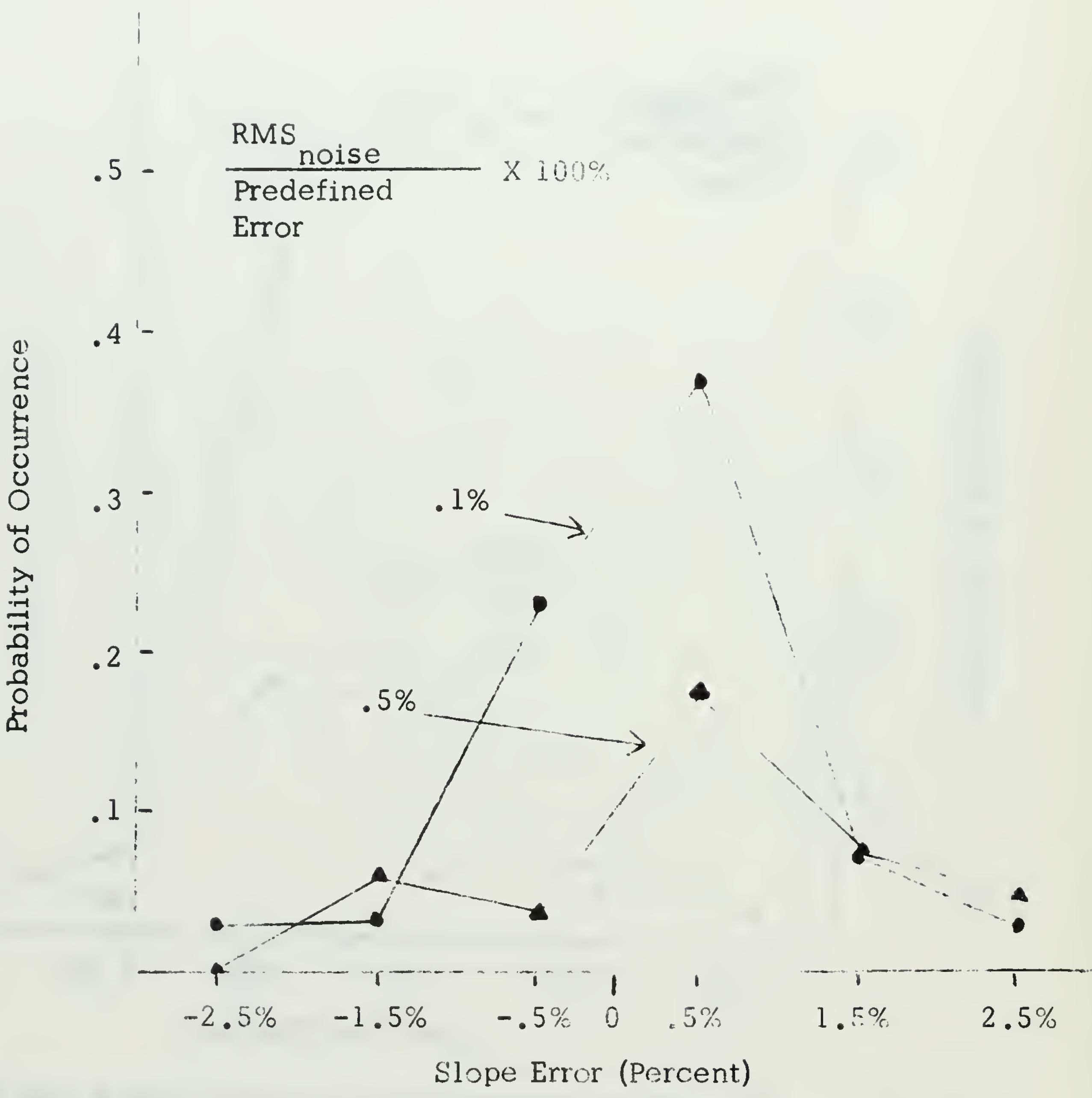


FIGURE 6.6f PROBABILITY DISTRIBUTION OF SLOPE ERROR FOR A PREDEFINED ERROR OF .1

ERR/E	Within ±.5%	Within ±1%	Within ±2%	Within ±3%	Within ±5%	Within ±10%	Within ±20%	Within ±25%
				E=.001				
1%	96%	100%	100%	100%	100%	100%	100%	100%
5%	66%	75%	86%	91%	94%	98%	100%	100%
10%	37%	55%	72%	76%	85%	94%	97%	100%
20%	30%	41%	57%	63%	73%	85%	97%	99%
				E=.005				
.5%	77%	91%	96%	98%	98%	100%	100%	100%
1%	67%	78%	91%	93%	95%	97%	100%	100%
2%	48%	63%	83%	85%	94%	97%	100%	100%
5%	19%	35%	51%	60%	74%	93%	97%	100%
10%	11%	31%	42%	66%	76%	91%	97%	100%
				E=.01				
.5%	79%	88%	95%	96%	99%	100%	100%	100%
1%	47%	72%	84%	91%	96%	100%	100%	100%
2%	38%	51%	68%	81%	84%	95%	100%	100%
5%	7%	20%	43%	59%	74%	79%	91%	94%
10%	8%	13%	31%	40%	51%	64%	88%	91%
				E=.02				
.5%	55%	75%	84%	90%	95%	99%	100%	100%
1%	44%	58%	72%	77%	90%	96%	97%	100%
2%	16%	26%	43%	56%	69%	82%	90%	90%
5%	5%	15%	23%	36%	50%	64%	73%	76%

TABLE I - RESULTS OF COMPUTER SIMULATION

E=.05

ERR/E	Within ±.5%	Within ±1%	Within ±2%	Within ±3%	Within ±5%	Within ±10%	Within ±20%	Within ±25%
.25%	57%	62%	74%	79%	82%	93%	100%	100%
.5%	51%	54%	64%	73%	83%	94%	98%	99%
1%	20%	30%	38%	43%	59%	82%	93%	95%
2%	3%	8%	18%	24%	36%	60%	80%	83%
				E=.1				
.1%	60%	70%	77%	88%	91%	94%	98%	98%
.5%	22%	35%	40%	46%	56%	76%	88%	90%
1%	17%	23%	31%	35%	41%	59%	71%	75%

exceed the predefined error in a time interval t.

F. CONCLUSION

The preceeding results show that for the continuous secant compressor, the factor that affects the efficiency the most is the predefined error and not the RMS noise. The compressor depends twice as much on the predefined error as it does on the noise. There is an optimum point where every compressor should operate, depending on the specifications of the system. Finally, the conclusion is made that the distribution of the slope of $M_2(t)$ in the presence of noise is approximately gaussian.

In summary, Figures 6.7 and 6.8 show the final probability distribution as a function of slope error and predefined error.

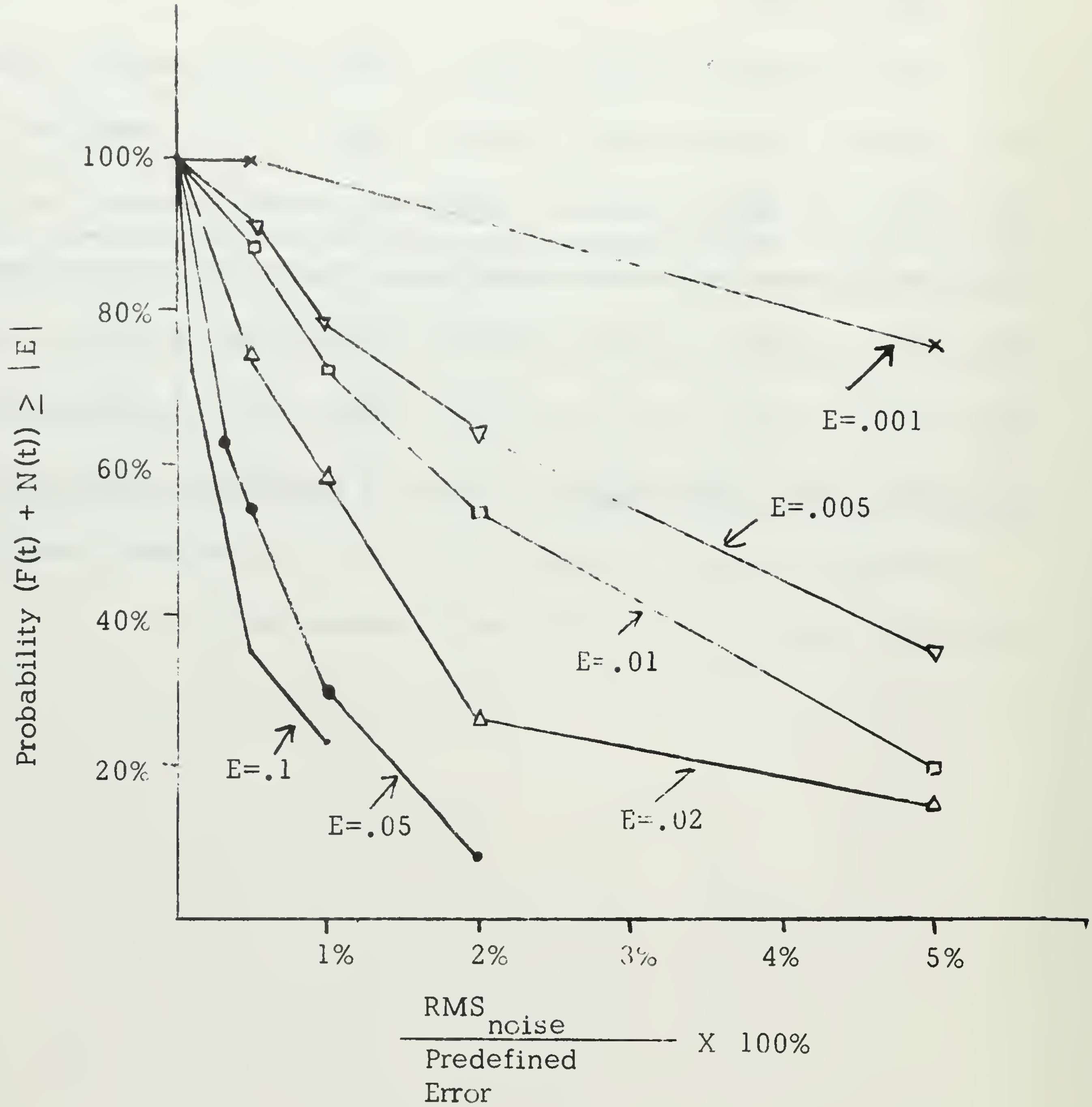


FIGURE 6.7 PROBABILITY DISTRIBUTION OF SLOPE PLUS NOISE TO WITHIN 1% OF TRUE SLOPE

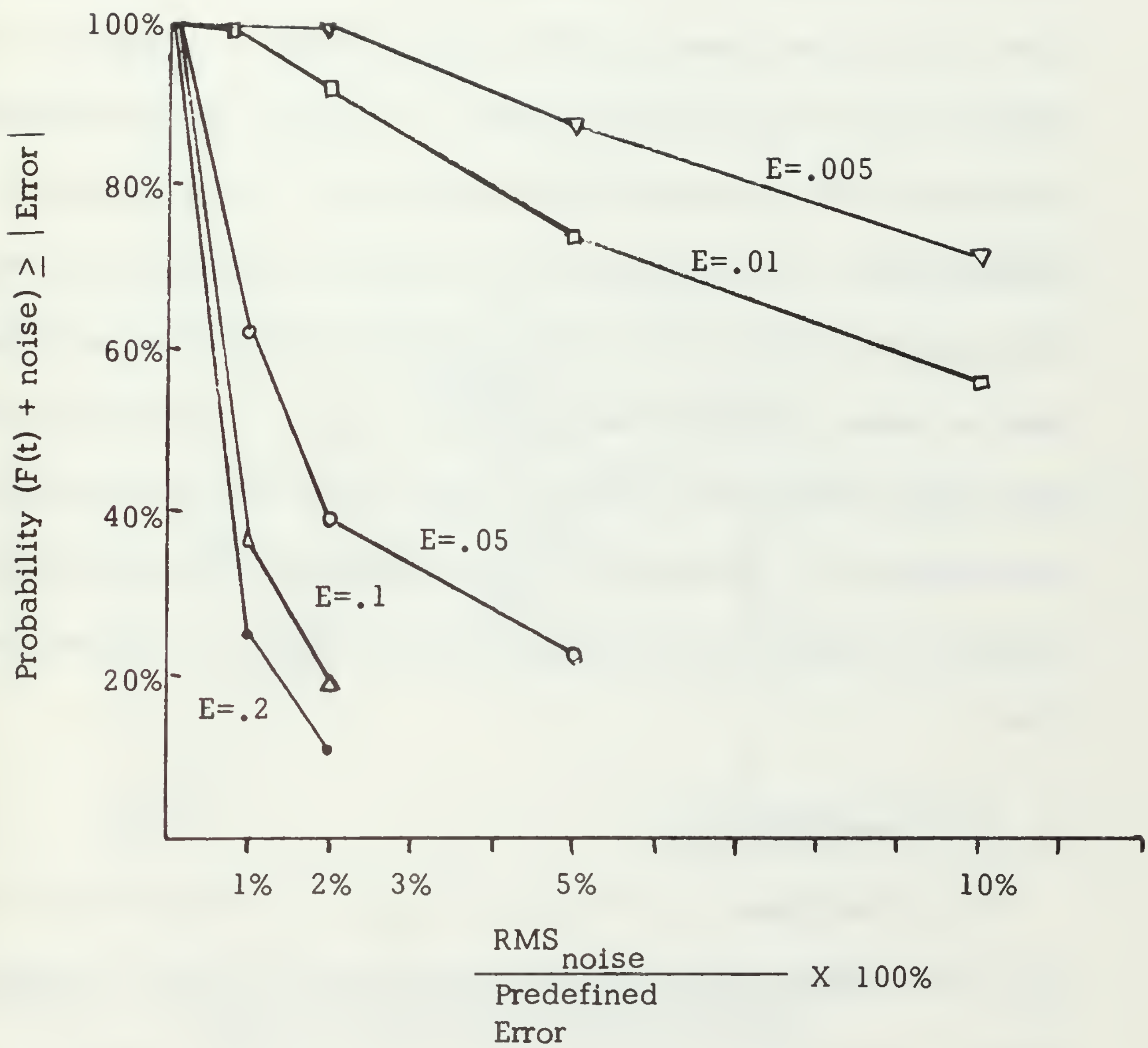


FIGURE 6.8 PROBABILITY DISTRIBUTION OF SIGNAL AND NOISE FOR A SLOPE ERROR OF .0001

VII. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

Interpolation techniques give a better compression ratio than prediction techniques, but the increased complexity of interpolators make them difficult to implement. The continuous secant compressor, which determines the optimum sampling interval before sampling, proved to be very efficient even in the presence of noise. A straight-line approximation is the basis for the continuous secant compressor. In the presence of white gaussian noise, the distribution of the slope of the straight line had a fairly gaussian distribution. The distribution of the slope was dependent upon the slope of the input signal at each sample. A completely random input signal composed of the sum of eight sine waves gave a good gaussian distribution. Although the white noise had affected the slope distribution, the probability distribution depended largely on the predefined error.

B. RECOMMENDATIONS

The question that now remains is what is the probability that the signal plus noise will exceed the predefined error within a specified time interval. The graphic representation of this statement appears in Fig. 7.1. Although the analytical results were not formulated, the experimental results showed that the probability distribution was gaussian for a completely random input. Since E remains constant from run to run, the

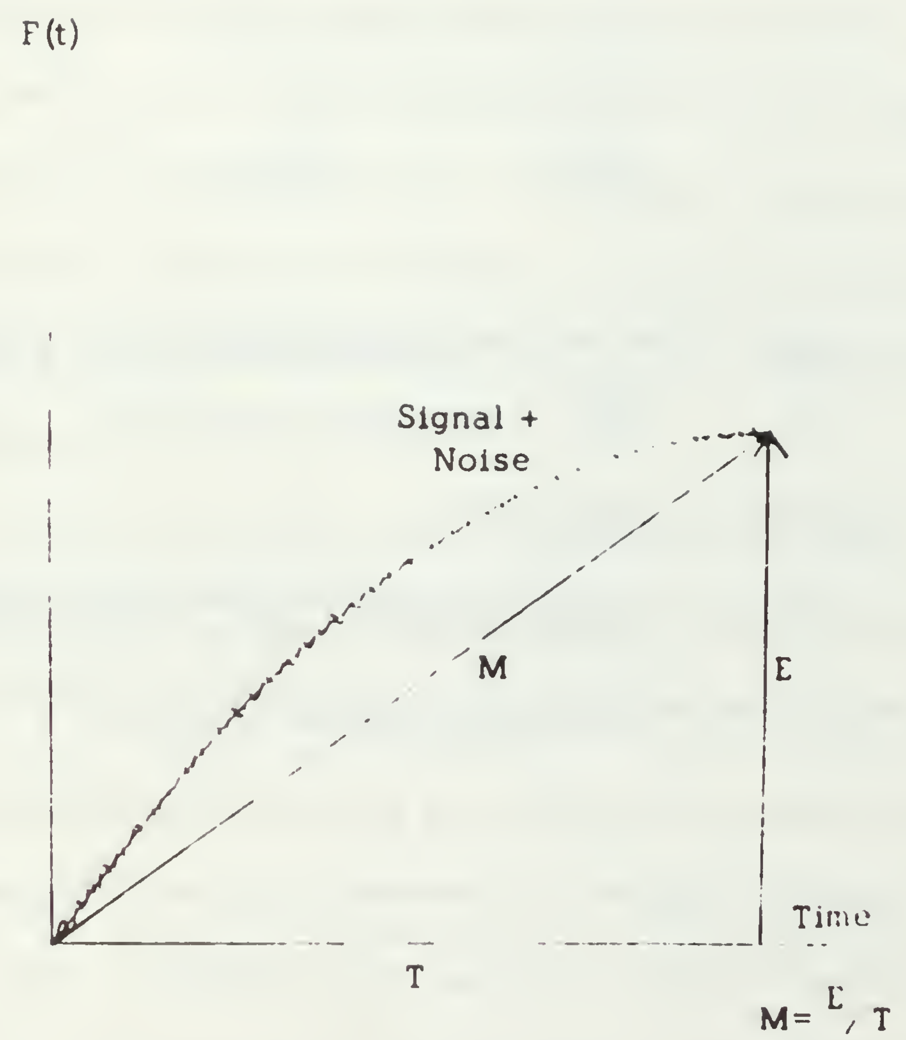


FIGURE 7.1 PROBABILITY OF EXCEEDING THE PREDEFINED ERROR IN A SPECIFIED TIME INTERVAL T

probability distribution of the slope can be directly related to the probability distribution on the time axis. This probability distribution of $t-t_o$, the time when the signal plus noise exceeds the predefined error E, becomes a good measure of slope distribution. Assuming that the input signal contains white noise, the equation for the slope becomes

$$M_{2n}(t) = \frac{F(t) + N(t) - [F(t_o) + N(t_o)]}{t-t_o} \tag{7-1}$$

$$= \underbrace{\frac{F(t) - F(t_o)}{t-t_o}}_{\uparrow \; M_2(t)} + \underbrace{\frac{N(t) - N(t_o)}{t-t_o}}_{\uparrow \; \text{ERROR}}$$

where

$M_{2n}(t)$ = slope of $M_2(t)$ with noise added to the signal

$N(t)$ = white noise added to the signal

$F(t)$ = input signal

$t-t_o$ = time when the signal plus noise exceeds the predefined error, optimum sampling interval

t_o = beginning of the time interval

As stated earlier in this thesis, as the predefined error is increased the optimum sampling interval is increased. Equation 7-1 shows that by increasing the time interval, the slope error is reduced. This agrees with the experimental results obtained in Chapter VI. The input signal also affects the sampling interval. A random input signal should have a shorter sampling interval than a fairly constant input signal. Equation 7-1 also shows that the slope error distribution is dependent upon the input noise. The noise at the beginning of the interval is subtracted from the noise at the end of the interval. If $N(t) = N(t_o)$ then the slope error would be zero but the magnitude of each sample would be in error by the amount N(t). If the noise is sufficiently small then the resultant slope error should be sufficiently small.

There is an optimum operating point for each system. The predefined error should be made as high as possible and still be within the system error specifications for reconstruction of the input signal, and at the same time, the RMS noise should be kept to a minimum. A noisy environment would require a high sampling interval to give a small error.

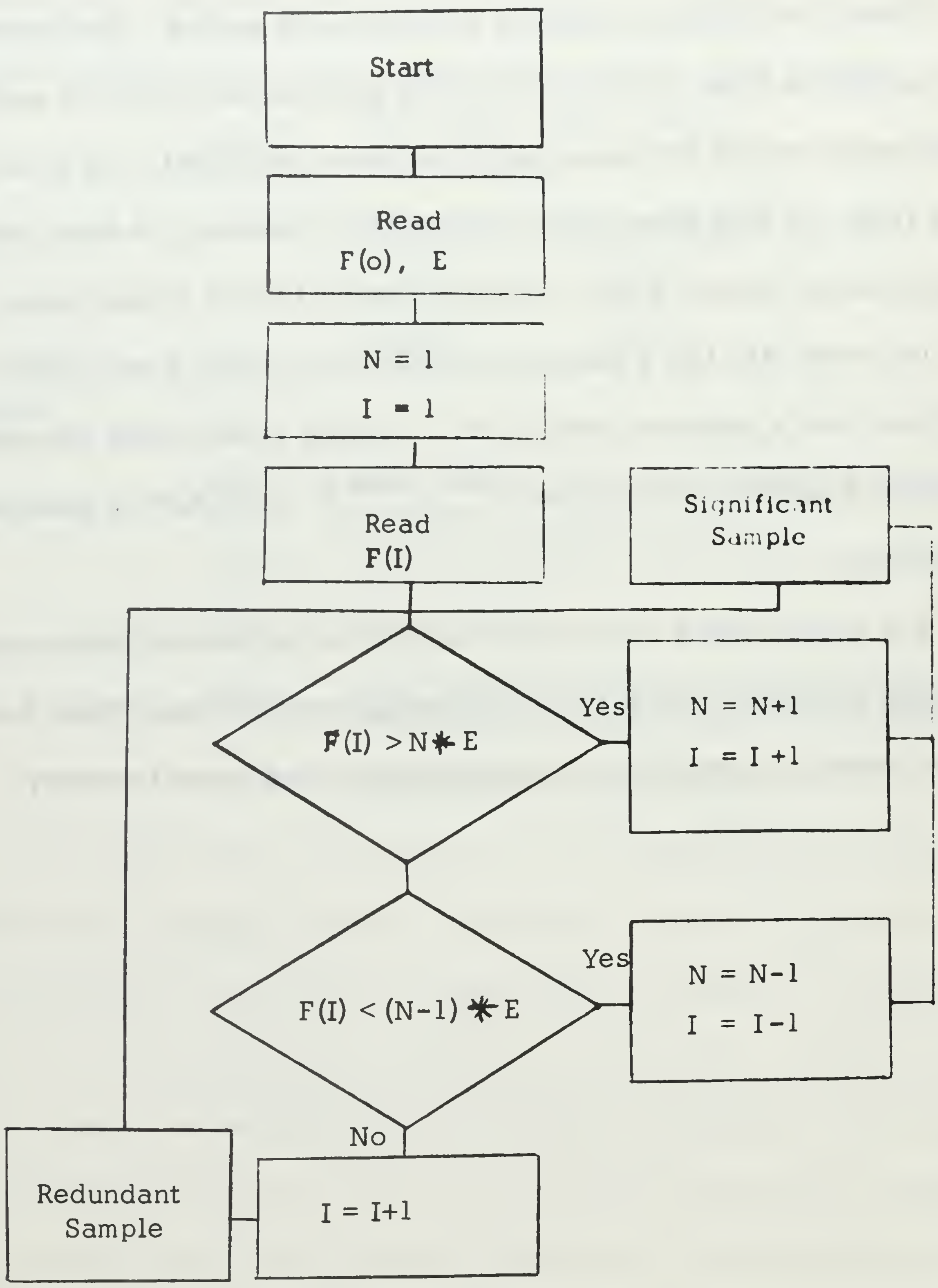
The noise N(t) has a gaussian distribution, and as a result N(t) - $N(t_0)$ also has a gaussian distribution. In order to prove that the slope error has a gaussian distribution, $\dfrac{N(t) - N(t_0)}{t - t_0}$ must have a gaussian distribution.

It is recommended that studies continue in the field of data compression due to the fact that it is an increasingly valuable tool which has future potential in the field of communications and space telemetry.

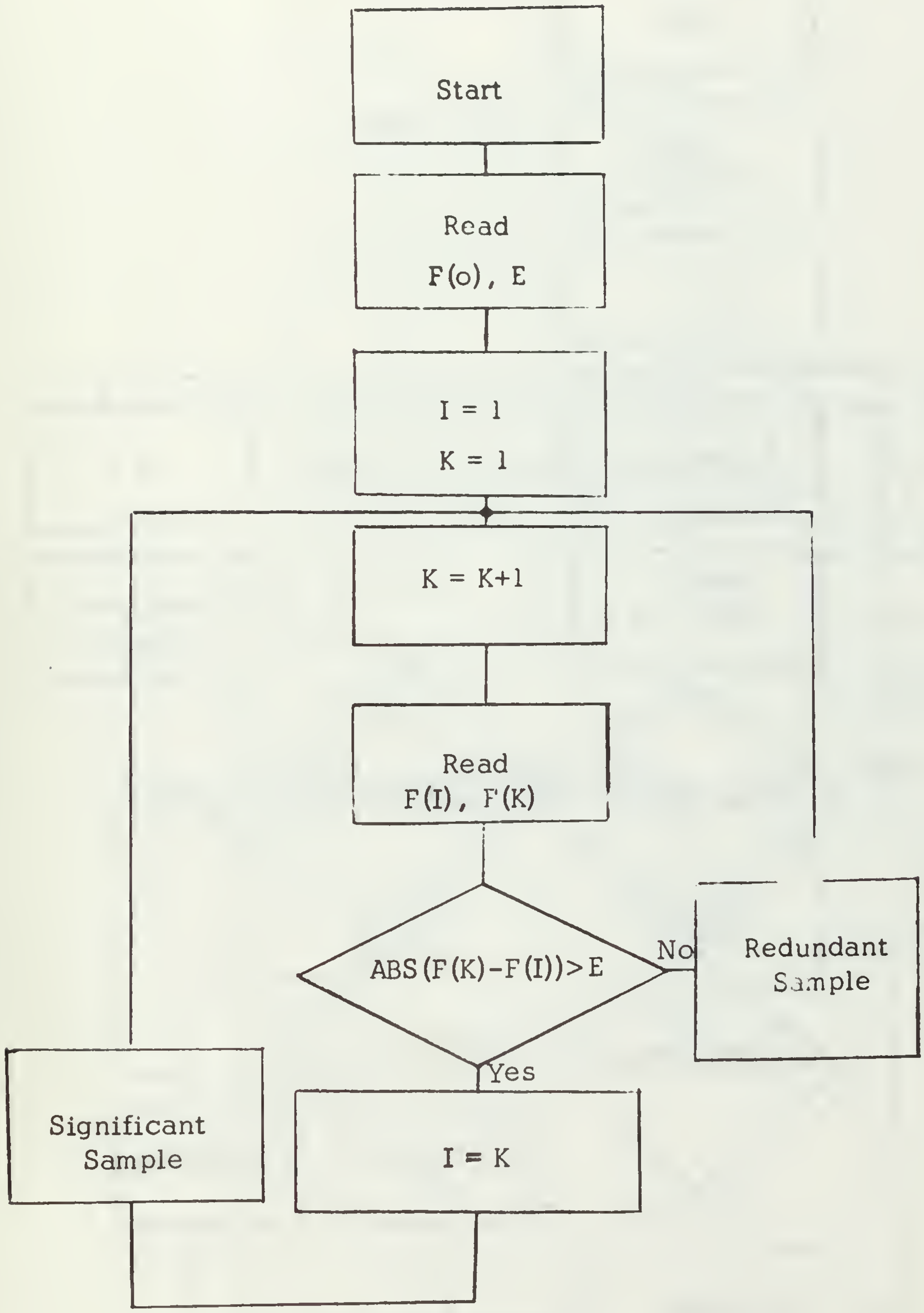
APPENDIX A

FLOW GRAPHS

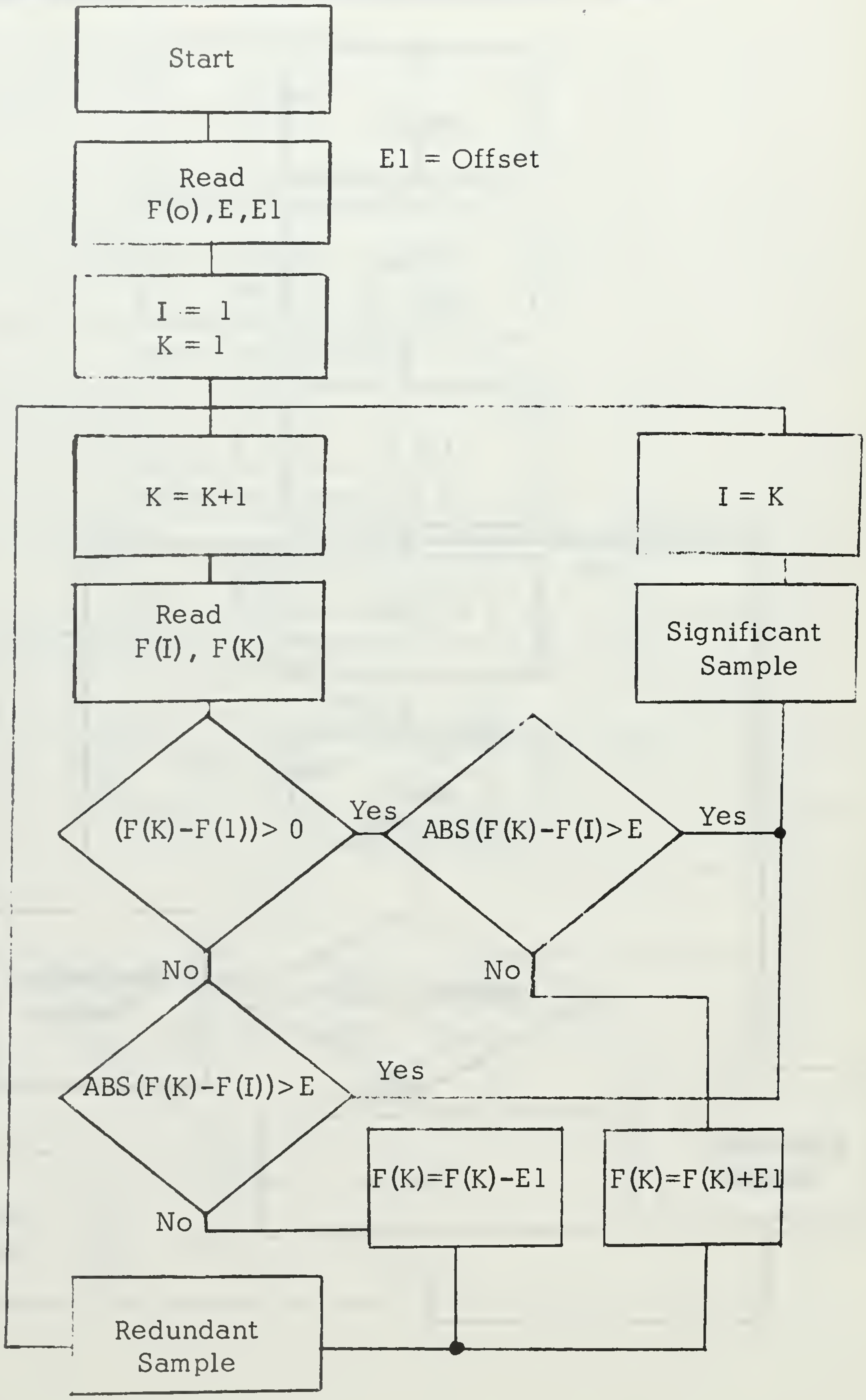
ZERO-ORDER PREDICTOR FIXED APERTURE



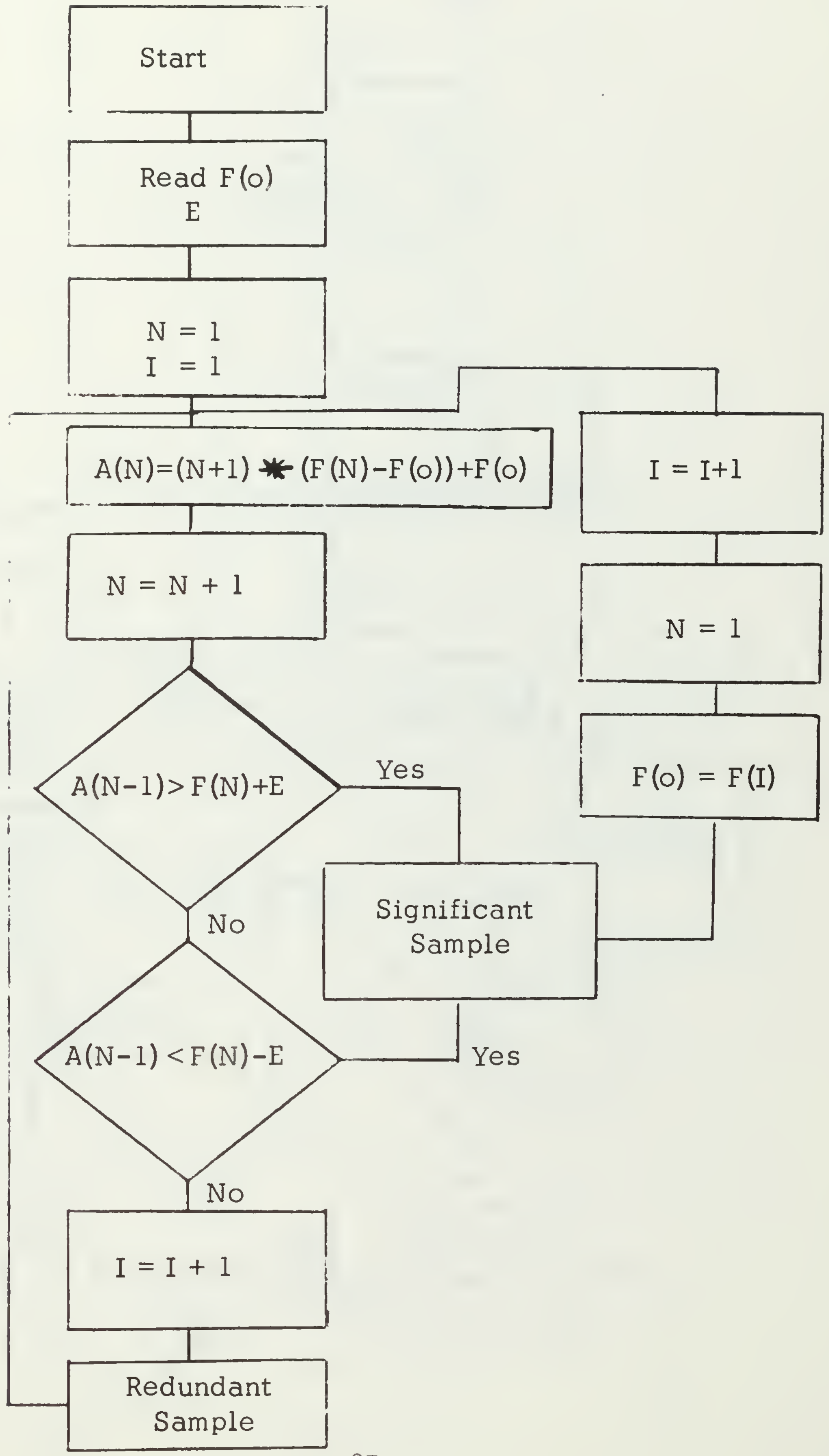
ZERO-ORDER PREDICTOR FLOATING APERTURE



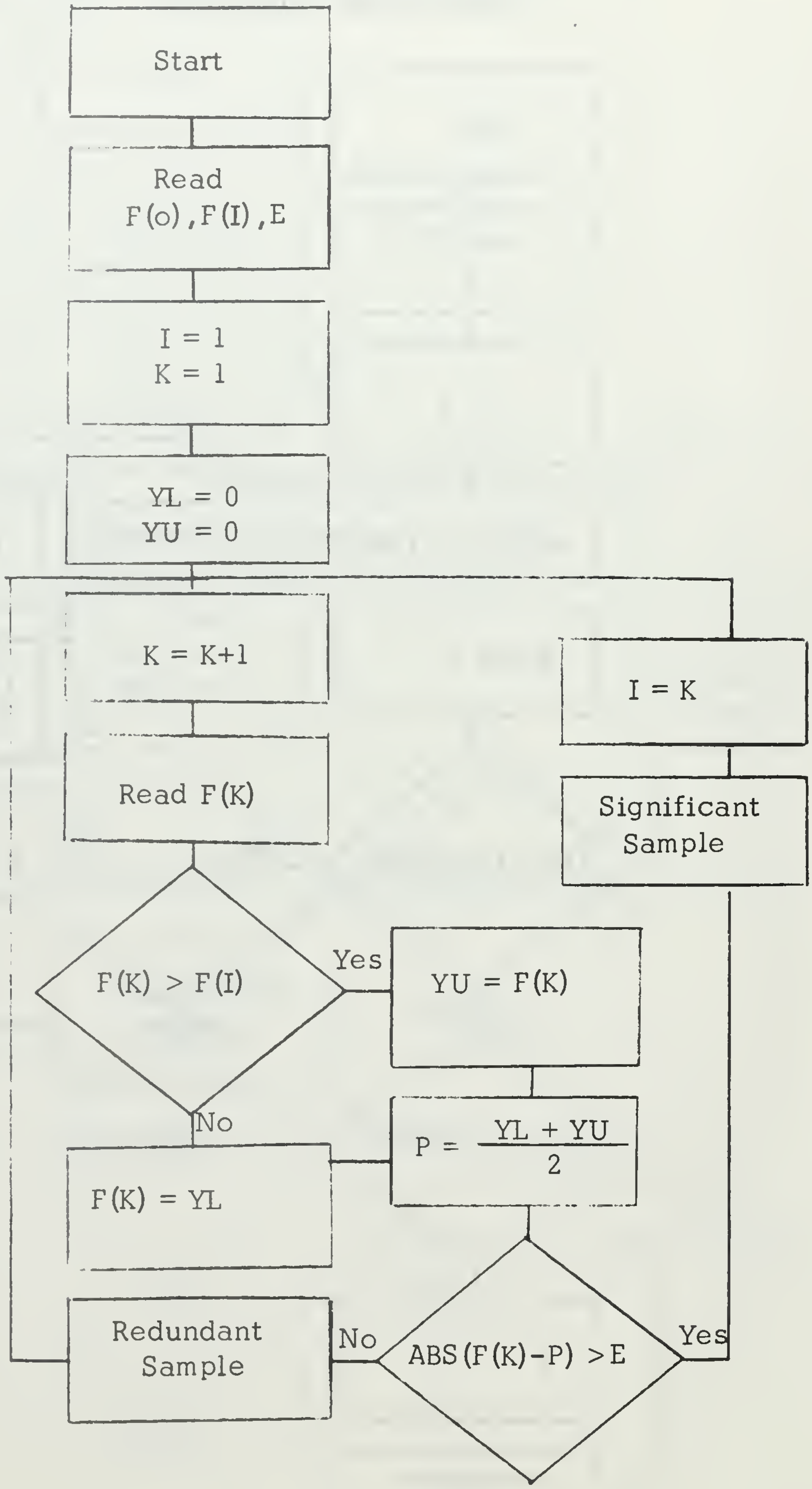
ZERO-ORDER OFFSET PREDICTOR



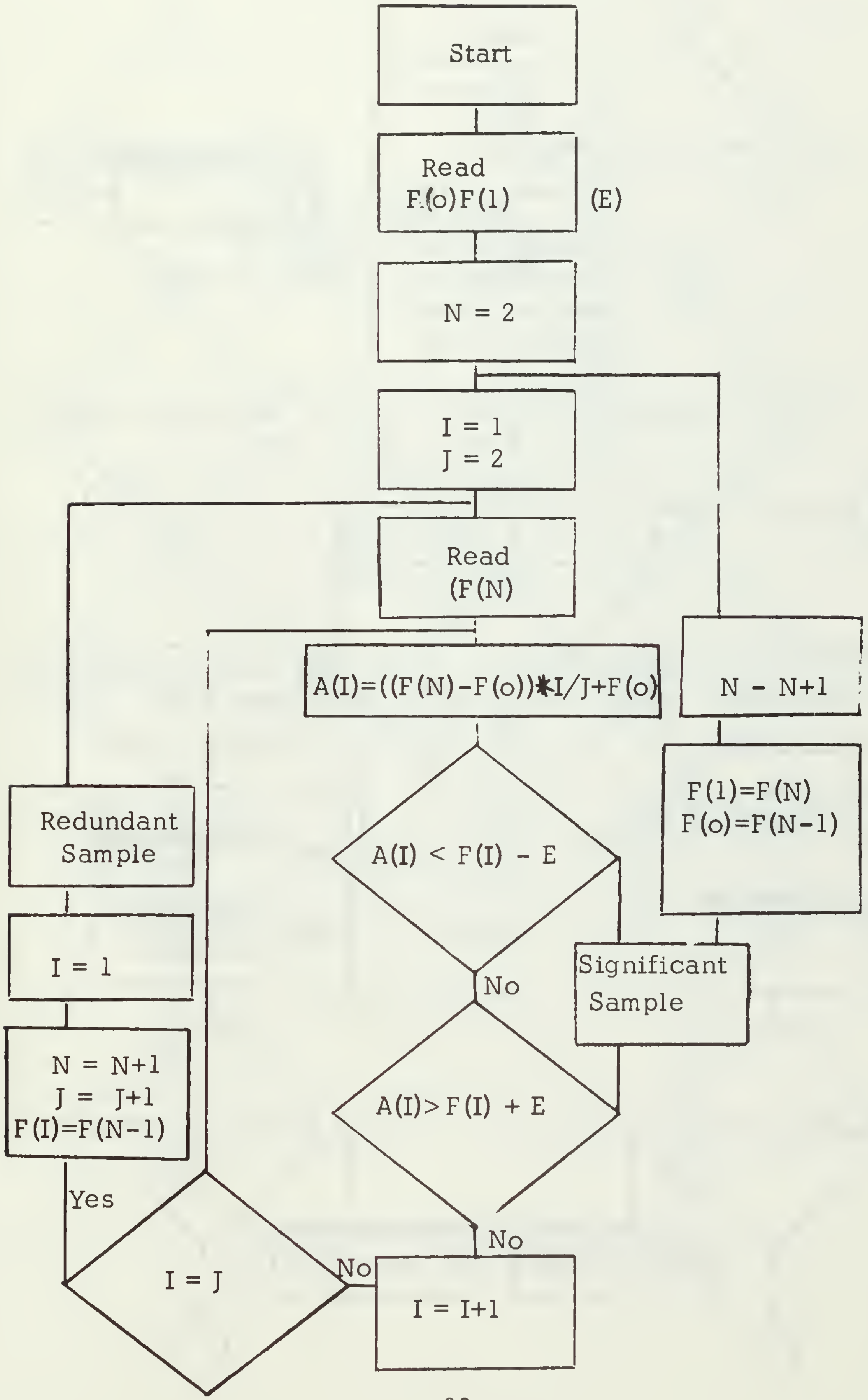
FIRST-ORDER PREDICTOR

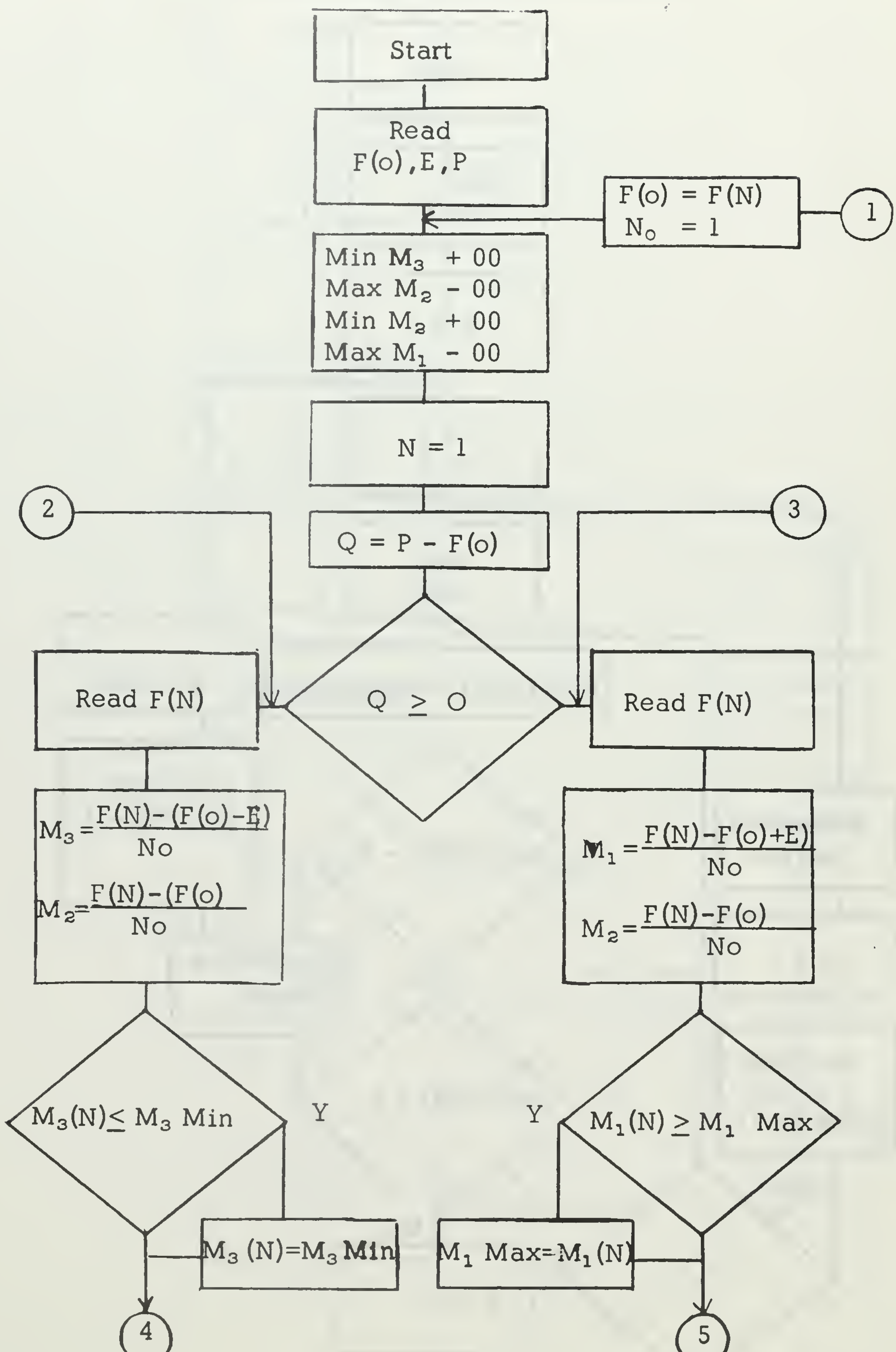


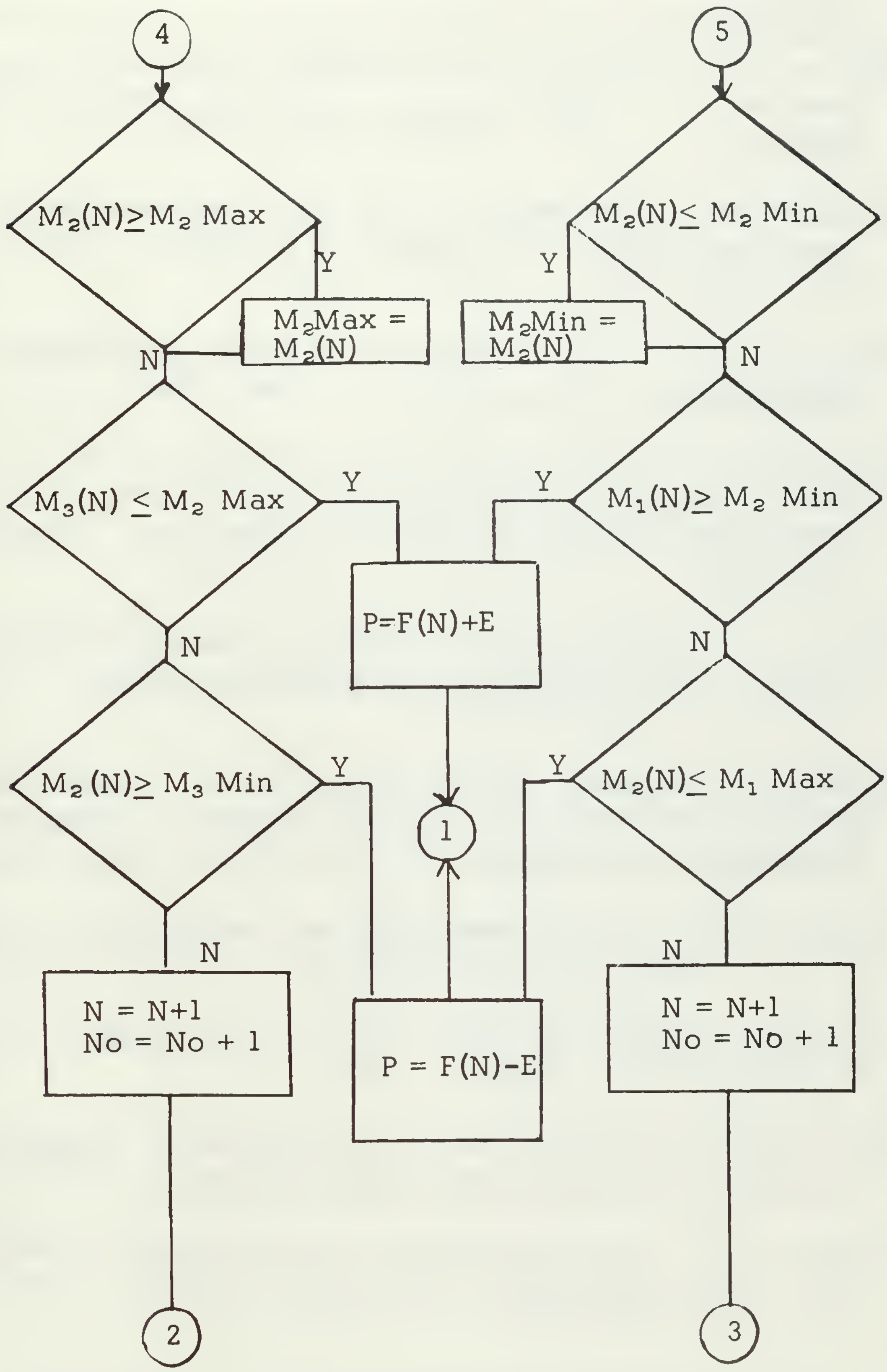
ZERO-ORDER INTERPOLATOR



FIRST-ORDER INTERPOLATOR




Start
Read
F(o),E,P
F(o) = F(N)
N_o = 1
1
Min M_3 + 00
Max M_2 - 00
Min M_2 + 00
Max M_1 - 00
N = 1
2
3
Q = P - F(o)
Read F(N)
Q ≥ O
Read F(N)
$M_3 = \frac{F(N)-(F(o)-E)}{No}$
$M_2 = \frac{F(N)-(F(o)}{No}$
$M_1 = \frac{F(N)-F(o)+E)}{No}$
$M_2 = \frac{F(N)-F(o)}{No}$
$M_3(N) \leq M_3$ Min
Y
Y
$M_1(N) \geq M_1$ Max
M_3 (N)=M_3 Min
M_1 Max=M_1(N)
4
5



CONTINUOUS SECANT METHOD

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13. ABSTRACT

A study of data compression techniques involving linear interpolation and linear prediction showed that redundancy is a problem that can be significantly reduced by various polynomial approximations. A more recent compressor, the continuous secant compressor which determines the optimum sampling interval prior to sampling, was found to be the most efficient compressor examined. The continuous secant compressor bases its reduction technique on a straight-line approximation. Data compression results when the system in question does not occupy its entire bandwidth. The addition of white noise over the entire bandwidth was found to reduce the efficiency of the continuous secant compressor by only a small amount. The probability distribution of the straight-line approximation in the presence of noise had a gaussian distribution and a relatively small standard deviation.

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Redundancy						
Telemetry						
Compression						
Predictors						
Interpolators						